

Chapter 2 Reasoning and Proof

Lesson 2.1 Notes Inductive Reasoning and Conjecture

1 Make Conjectures Inductive reasoning is reasoning that uses a number of specific examples to arrive at a conclusion. When you assume that an observed pattern will continue, you are applying inductive reasoning. A concluding statement reached using inductive reasoning is called a **conjecture**.

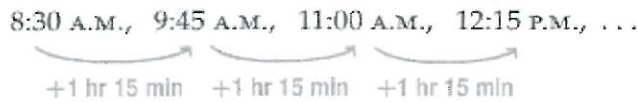


Example 1 Patterns and Conjecture

Write a conjecture that describes the pattern in each sequence. Then use your conjecture to find the next item in the sequence.

- a. Movie show times: 8:30 A.M., 9:45 A.M., 11:00 A.M., 12:15 P.M., ...

Step 1 Look for a pattern.



Step 2 Make a conjecture.

The show time is 1 hour and fifteen minutes greater than the previous show time. The next show time will be 12:15 P.M. + 1:15 or 1:30 P.M.

- b.
- 4 10 18 28 40 ...



The numbers increase by 6, 8, 10, and 12.

Step 2 The next figure will increase by 12 + 2 or 14 segments. So, the next figure will have 40 + 14 or 54 segments.

CHECK Draw the next figure to check your conjecture ✓



54

Guided Practice

Write a conjecture that describes the pattern in each sequence. Then use your conjecture to find the next item in the sequence.

- 1A. Follow-up visits: Dec., May, Oct., Mar., ...

Aug Follow up visits are every 5 months, so, next visit will be in August.

- 1B. 10, 4, -2, -8, ...

-6 -6 -6

The numbers decreases by 6, so, the next number will be $-8 - 6 = -14$.

Example 2 Algebraic and Geometric Conjectures

Make a conjecture about each value or geometric relationship. List or draw some examples that support your conjecture.

a. the sum of two odd numbers

Step 1 List examples.

$$1 + 3 = 4 \quad 1 + 5 = 6 \quad 3 + 5 = 8 \quad 7 + 9 = 16$$

Step 2 Look for a pattern.

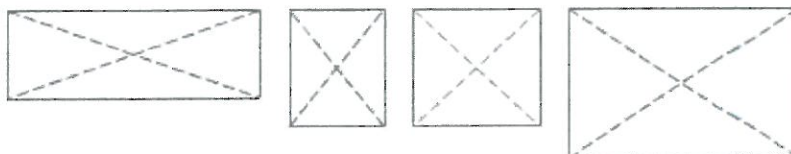
Notice that the sums 4, 6, 8, and 16 are all even numbers.

Step 3 Make a conjecture.

The sum of two odd numbers is an even number.

b. segments joining opposite vertices of a rectangle

Step 1



Step 2 Notice that the segments joining opposite vertices of each rectangle appear to have the same measure. Use a ruler or compass to confirm this.

Step 3 Conjecture: the segments joining opposite vertices of a rectangle are congruent.

Guided Practice

2A. the sum of two even numbers

Ex: $2 + 4 = 6$
 $4 + 10 = 14$
 $16 + 16 = 32$ } The sum of 2 even numbers is even.

2B. the relationship between AB and EF , if $AB = CD$ and $CD = EF$

$\overline{AB} \cong \overline{CD}$
 $\overline{CD} \cong \overline{EF}$
 $\overline{AB} \cong \overline{EF}$

Since $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$,
 $\overline{AB} \cong \overline{EF}$

2C. the sum of the squares of two consecutive natural numbers

Ex: $1^2 + 2^2 = 5$
 $3^2 + 4^2 = 25$
 $5^2 + 6^2 = 25 + 36 = 61$ } The sum of the squares of two consecutive natural numbers is odd.

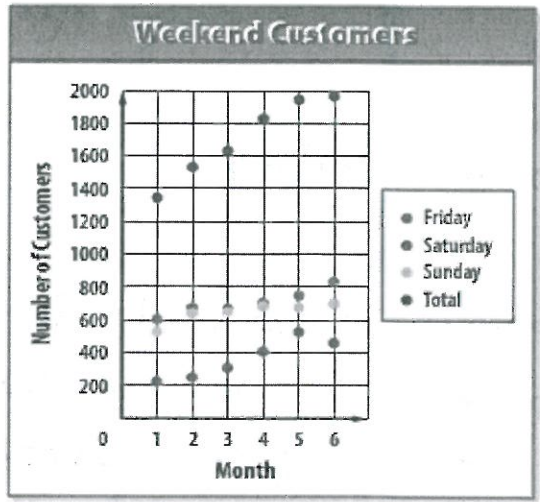
Real-World Example 3 Make Conjectures from Data

BUSINESS The owner of a hair salon collected data on the number of customers her salon had each Friday, Saturday, and Sunday for 6 months to decide whether she should increase the number of stylists working each weekend. The data she collected are shown below.

Number of Customers on the Weekend						
Day	Month 1	Month 2	Month 3	Month 4	Month 5	Month 6
Friday	225	255	321	406	540	450
Saturday	603	658	652	712	746	832
Sunday	552	635	642	692	685	705
Total	1380	1548	1615	1810	1971	1987

a. Make a statistical graph that best displays the data.

Since you want to look for a pattern over time, use a scatter plot to display the data. Label the horizontal axis with the months and the vertical axis with the number of customers. Plot each set of data using a different color, and include a legend.



b. Make a conjecture based on the data, and explain how this conjecture is supported by your graph.

Look for patterns in the data. The number of customers on each day usually increases each month, and the total number of customers increases every single month.

Survey data supports a conjecture that the amount of business on the weekends has increased, so the owner should schedule more stylists to work on those days.

Guided Practice

3. **POSTAGE** The table at the right shows the price of postage for the years 1982 through 2009.

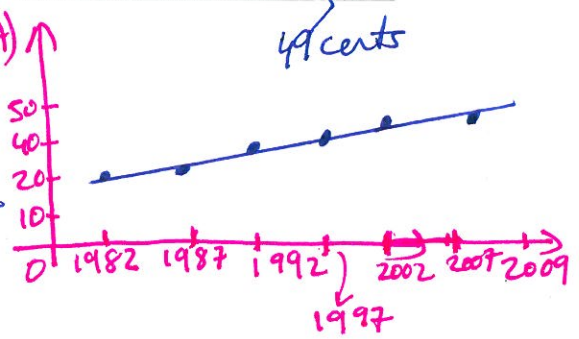
Year	Rate (cents)
1982	20
1987	22
1992	29
1997	32
2002	37
2007	41
2009	44

A. Make a statistical graph that best displays the data.

B. Predict the postage rate in 2015 based on the graph. *49 cents*

C. Does it make sense that the pattern of the data will continue over time? If not, how will it change? Explain your reasoning.

Yes the trend is reasonable because the price of the postage will probably continue to increase over time.



2 Find Counterexamples To show that a conjecture is true for all cases, you must prove it. It takes only one false example, however, to show that a conjecture is not true. This false example is called a **counterexample**, and it can be a number, a drawing, or a statement.

VocabularyLink

Counterexample

Everyday Use The prefix *counter-* means the opposite of.

Math Use A counterexample is the opposite of an example.

Example 4 Find Counterexamples

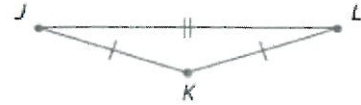
Find a counterexample to show that each conjecture is false.

a. If n is a real number, then $n^2 > n$.

When n is 1, the conjecture is false, since $1^2 \not> 1$.

b. If $JK = KL$, then K is the midpoint of \overline{JL} .

When J , K , and L are noncollinear, the conjecture is false. In the figure, $JK = KL$, but K is not the midpoint of \overline{JL} .



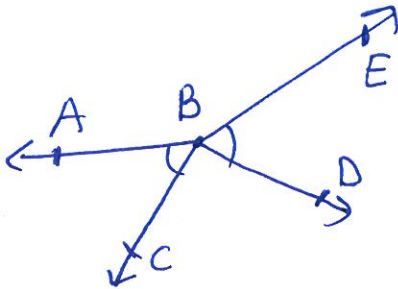
Guided Practice

4A. If n is a real number, then $-n$ is a negative number.

Ex: $n = -4 \rightarrow -n = -(-4) = 4$

4B. If $\angle ABC \cong \angle DBE$, then $\angle ABC$ and $\angle DBE$ are vertical angles.

When points A , B and D are noncollinear and points E , B and C are noncollinear, the conjecture is false. In the figure, $\angle ABC \cong \angle DBE$, but $\angle ABC$ and $\angle DBE$ are not vertical angles.

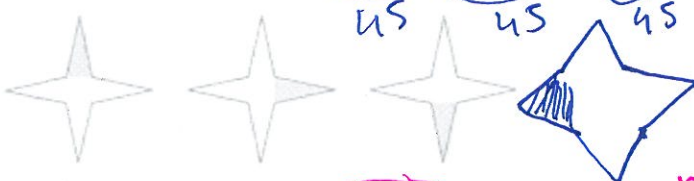


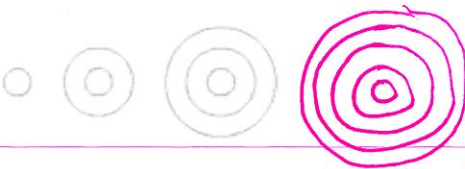
Homework: Complete all questions on this sheet by 9/29 beginning of the lesson

Write a conjecture that describes the pattern in each sequence. Then use your conjecture to find the next item in the sequence.

1. Costs: \$4.50, \$6.75, \$9.00, ... Each cost is \$2.25 more than the previous. \$11.25

2. Appointment times: 10:15 A.M., 11:00 A.M., 11:45 A.M., ... 12:30pm Each time is 45 minutes later than the previous time; 12:30pm

3.  In each figure, the shading moves to the next point clockwise.

4.  In each figure, the pattern has an additional circle around the outside.

5. 3, 3, 6, 9, 15, ... Each element in the pattern is the sum of the previous two elements; 24.

6. 2, 6, 14, 30, 62, ... Each element is two more than two times the previous element; 126

Make a conjecture about each value or geometric relationship.

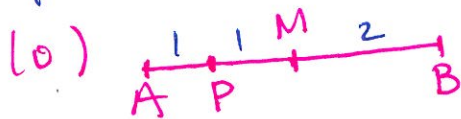
7. the product of two even numbers The product of 2 even numbers is an even number

8. the relationship between a and b if $a + b = 0$ $3 + (-3) = 0$ } a and b are additive inverses.
 $5 + (-5) = 0$ }

9. the relationship between the set of points in a plane equidistant from point A

10. the relationship between \overline{AP} and \overline{PB} if M is the midpoint of \overline{AB} and P is the midpoint of \overline{AM}

9) The set of points in a plane equidistant from point A is a circle.



\overline{PB} is three times as long as \overline{AP} .

READ ALL PARTS AND WRITE DOWN THE VOCABULARY in YOUR NOTEBOOK! COMPLETE ALL PRACTICE QUESTIONS! KEEP IT IN YOUR BINDER! IF YOU LOOSE IT = 0 GRADE!!!

