Name Hanci Date________ Period________

5.2 Verifying Trigonometric Identities

You will prove that both sides of the equation are equal for all values of the variables on both sides.

1)Verify an Identity by Simplifying One Side of the Equation: Start from the most complicated side

Examples: Verify each of the following identities.

1. **Verify that** $\frac{\csc^2 x - 1}{\csc^2 x} = \cos^2 x$.

*Start with the left side

*Remember $\csc^2 x = 1 + \cot^2 x$
$-1 \quad -1$

* $\csc^2 x - 1 = \cot^2 x$

*Remember $\frac{1}{\csc x} = \sin x$

$$\frac{\csc^2 x - 1}{\csc^2 x} = \frac{\cot^2 x}{\csc^2 x}$$

$$= \cot^2 x \sin^2 x$$

$$= \left(\frac{\cos^2 x}{\cancel{\sin^2} x}\right) \times \cancel{\sin^2} x$$

$$= \boxed{\cos^2 x = \cos^2 x}$$

Yes, left-side is equal to right-side :VERIFIED!

2. $\sec^2 \theta \cot^2 \theta - 1 = \cot^2 \theta$

*Begin with the left-side

$$\sec^2\theta \cot^2\theta - 1 = \cot^2\theta$$

$$\frac{1}{\cancel{\cos^2\theta}} \times \frac{\cancel{\cos^2\theta}}{\sin^2\theta} - 1 \stackrel{?}{=} \cot^2\theta$$

$$\frac{1}{\sin^2\theta} - 1 \stackrel{?}{=} \cot^2\theta$$

$$\csc^2\theta - 1 \stackrel{?}{=} \cot^2\theta$$

*Pythagorean Identity

replace

$$\cot^2\theta + 1 = \csc^2\theta$$
$$-1 \qquad -1$$

$$\cot^2\theta = \csc^2\theta - 1$$

$$\boxed{\cot^2\theta = \cot^2\theta}$$

3. $\tan^2 \alpha = \cancel{\sec\alpha} \csc \alpha \tan \alpha - 1$

*Start with right-side

$$\tan^2\alpha \stackrel{?}{=} \frac{1}{\cos\alpha} \times \frac{1}{\cancel{\sin\alpha}} \times \frac{\cancel{\sin\alpha}}{\cos\alpha} - 1$$

$$\tan^2\alpha \stackrel{?}{=} \frac{1}{\cos^2\alpha} - 1$$

$$\tan^2\alpha \stackrel{?}{=} \sec^2\alpha - 1$$

replace

*Pythagorean Identity

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\boxed{\tan^2\theta = \sec^2\theta - 1}$$

$$\boxed{\tan^2\alpha = \tan^2\alpha}$$

4. Verify that $\frac{\sin\alpha}{1-\cos\alpha} = \csc\alpha + \cot\alpha$.

*Left-side can be considered more complicated due to the fraction.

So, check the left-side first.

$$\frac{\sin\alpha}{1-\cos\alpha}\frac{(1+\cos\alpha)}{(1+\cos\alpha)} = \frac{\sin\alpha+\sin\alpha\cos\alpha}{1-\cos^2\alpha}$$

Pythagorean Identity

$\sin^2\alpha+\cos^2\alpha=1$

$\sin^2\alpha = 1-\cos^2\alpha$

$$= \frac{\cancel{\sin}\alpha(1+\cos\alpha)}{\cancel{\sin^2}\alpha}$$

$$= \frac{1+\cos\alpha}{\sin\alpha}\frac{(\csc\alpha)}{(\csc\alpha)}$$

$$= \frac{\csc\alpha+\cos\alpha\csc\alpha}{\cancel{\sin\alpha}\cdot\frac{1}{\cancel{\sin\alpha}}}$$

$$= \csc\alpha+\cos\alpha\csc\alpha = \csc\alpha + \underbrace{\cos\alpha\cdot\frac{1}{\sin\alpha}}_{\cot\alpha} = \boxed{\csc\alpha+\cot\alpha}$$

5. Verify that $\cot\theta\sec\theta\csc^2\theta - \cot^3\theta\sec\theta = \csc\theta$.

First, simplify the right-side by using factoring

$$\cot\theta\sec\theta\csc^2\theta - \cot^3\theta\sec\theta = \cot\theta\sec\theta(\underbrace{\csc^2\theta-\cot^2\theta}_{=1})$$

Pythagorean Identity

$\cot^2\theta+1=\csc^2\theta$

$1=\csc^2\theta-\cot^2\theta$

$$= \cot\theta\sec\theta\times1$$

$$= \left(\frac{\cancel{\cos\theta}}{\sin\theta}\right)\times\left(\frac{1}{\cancel{\cos\theta}}\right)$$

$$= \frac{1}{\sin\theta}$$

$$= \boxed{\csc\theta}$$

2)Verify an Identity by Working Each Side Separately: You need to simplify both sides and compare the results to see if you get the same answer.

1. Verify that $\frac{\tan^2 x}{1+\sec x} = \frac{1-\cos x}{\cos x}$. Simplify each side seperately

Left side: multiply numerator & denominator by the conjugate of $(1+\sec x)$

$$\frac{\tan^2 x}{1+\sec x}\cdot\frac{(1-\sec x)}{(1-\sec x)} = \frac{\tan^2 x - \tan^2 x\sec x}{1-\sec^2 x} = \frac{\cancel{\tan^2 x}(1-\sec x)}{-\cancel{\tan^2 x}}$$

$$= \frac{(1-\sec x)}{-1}$$

$$= -1 + \sec x = \sec x - 1$$

Right side: start separating the fraction

$$\frac{1-\cos x}{\cos x} = \frac{1}{\cos x} - \frac{\cos x}{\cos x} = \frac{1}{\cos x} - 1$$

$$= \sec x - 1$$ ✓

So, then compare right & left sides: $\boxed{\sec x - 1 = \sec x - 1}$ ✓

VERIFIED

2. Verify that $\sec^4 x - \sec^2 x = \tan^4 x + \tan^2 x$.

Left Side: start with factoring.

$$\sec^4 x - \sec^2 x = \sec^2 x(\sec^2 x - 1)$$

$$= \sec^2 x\tan^2 x$$

, use Pythagorean Identity

$\tan^2\theta + 1 = \sec^2\theta$

$\tan^2\theta = \sec^2\theta - 1$

Right side:

$$\tan^4 x + \tan^2 x = \tan^2 x(\tan^2 x + 1)$$

, $\tan^2\theta + 1 = \sec^2\theta$

$$= \tan^2 x\sec^2 x$$

Compare what you get for right and left sides;

$\boxed{\sec^2 x\tan^2 x = \sec^2 x\tan^2 x}$ ✓

verified