NAME ______________________ DATE ______________________ PERIOD __________

8.1 Introduction to Vectors

1 Vectors Many physical quantities, such as speed, can be completely described by a single real number called a *scalar*. This number indicates the *magnitude* or *size* of the quantity. A **vector** is a quantity that has both magnitude and *direction*. The velocity of a football is a vector that describes both the speed and direction of the ball.

Example 1 Identify Vector Quantities

State whether each quantity described is a *vector* quantity or a *scalar* quantity.

a. a boat traveling at 15 miles per hour

This quantity has a magnitude of 15 miles per hour, but no direction is given. Speed is a scalar quantity.

b. a hiker walking 25 paces due west

This quantity has a magnitude of 25 paces and a direction of due west. This directed distance is a vector quantity.

c. a person's weight on a bathroom scale

Weight is a vector quantity that is calculated using a person's mass and the downward pull due to gravity. (Acceleration due to gravity is a vector.)

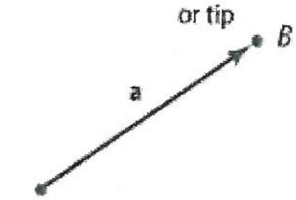
GuidedPractice

1A. a car traveling 60 miles per hour 15° east of south vector

1B. a parachutist falling straight down at 12.5 miles per hour vector

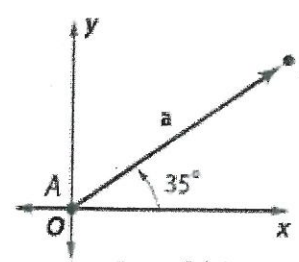
1C. a child pulling a sled with a force of 40 newtons scalar

A vector can be represented geometrically by a directed line segment, or arrow diagram, that shows both magnitude and direction. Consider the directed line segment with an **initial point** A (also known as the *tail*) and **terminal point** B (also known as the *head* or *tip*) shown. This vector is denoted by $\overrightarrow{AB}$, $\vec{a}$, or **a**.

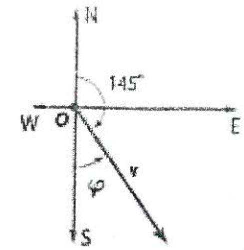


If a vector has its initial point at the origin, it is **in standard position**. The **direction** of a vector is the directed angle between the vector and the horizontal line that could be used to represent the positive x-axis. The direction of **a** is 35°.

The length of the line segment represents, and is proportional to, the **magnitude** of the vector. If the scale of the arrow diagram for **a** is 1 cm = 5 ft/s, then the magnitude of **a**, denoted |**a**|, is 2.6 × 5 or 13 feet per second.



The direction of a vector can also be given as a bearing. A **quadrant bearing** φ, or *phi*, is a directional measurement between 0° and 90° east or west of the north-south line. The quadrant bearing of vector **v** shown is 35° east of south or southeast, written S35°E.



A **true bearing** is a directional measurement where the angle is measured clockwise from north. True bearings are always given using three digits. So, a direction that measures 25° clockwise from north would be written as a true bearing of 025°.

StudyTip

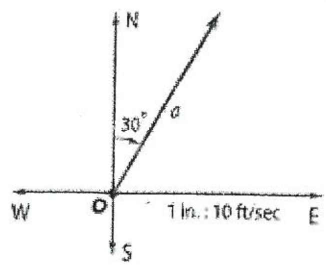
True Bearing When a degree measure is given without any additional directional components, it is assumed to be a true bearing. The true bearing of **v** is 145°.

Example 2 Represent a Vector Geometrically

Use a ruler and a protractor to draw an arrow diagram for each quantity described. Include a scale on each diagram.

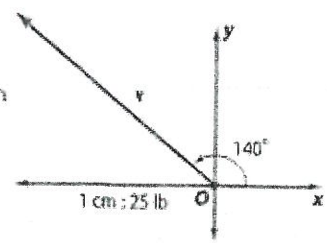
a. a = 20 feet per second at a bearing of 030°

Using a scale of 1 in. : 10 ft/sec, draw and label a 20 ÷ 10 or 2-inch arrow at an angle of 30° clockwise from the north.



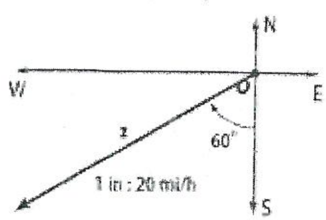
b. v = 75 pounds of force at 140° to the horizontal

Using a scale of 1 cm : 25 lb, draw and label a 75 ÷ 25 or 3-centimeter arrow in standard position at a 140° angle to the x-axis.

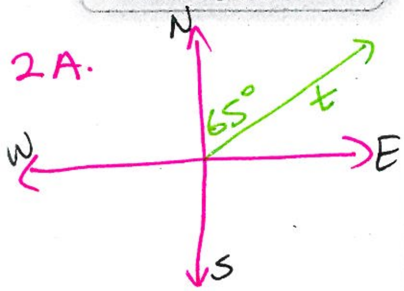


c. z = 30 miles per hour at a bearing of S60°W

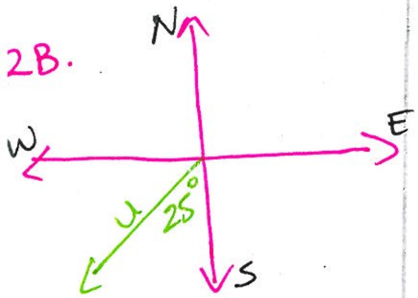
Using a scale of 1 in. : 20 mi/h, draw and label a 30 ÷ 20 or 1.5-inch arrow 60° west of south.



2A.



2B.



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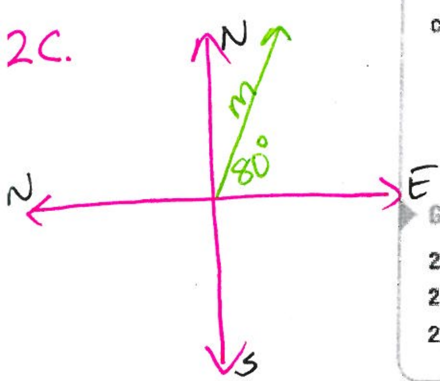

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2A. t = 20 feet per second at a bearing of 065°

2B. u = 15 miles per hour at a bearing of S25°E

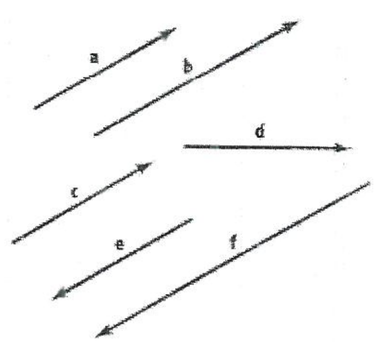
2C. m = 60 pounds of force at 80° to the horizontal

WatchOut!

Magnitude The magnitude of a vector can represent distance, speed, or force. When a vector represents velocity, the length of the vector does not imply distance traveled.

In your operations with vectors, you will need to be familiar with the following vector types.

- **Parallel vectors** have the same or opposite direction but not necessarily the same magnitude. In the figure, **a** ∥ **b** ∥ **c** ∥ **e** ∥ **f**.
- **Equivalent vectors** have the same magnitude and direction. In the figure, **a** = **c** because they have the same magnitude and direction. Notice that **a** ≠ **b**, since |**a**| ≠ |**b**|, and **a** ≠ **d**, since **a** and **d** do not have the same direction.
- **Opposite vectors** have the same magnitude but opposite direction. The vector opposite **a** is written −**a**. In the figure, **e** = −**a**.



When two or more vectors are added, their sum is a single vector called the **resultant**. The resultant vector has the same effect as applying one vector after the other. Geometrically, the resultant can be found using either the **triangle method** or the **parallelogram method**.

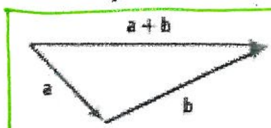
KeyConcept Finding Resultants

Triangle Method (Tip-to-Tail)

To find the resultant of a and b, follow these steps.

Step 1 Translate b so that the tail of b touches the tip of a.

Step 2 The resultant is the vector from the tail of a to the tip of b.



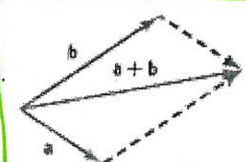
Parallelogram Method (Tail-to-Tail)

To find the resultant of a and b, follow these steps.

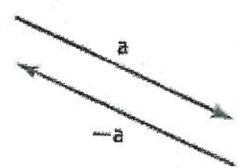
Step 1 Translate b so that the tail of b touches the tail of a.

Step 2 Complete the parallelogram that has a and b as two of its sides.

Step 3 The resultant is the vector that forms the indicated diagonal of the parallelogram.



When you add two opposite vectors, the resultant is the **zero vector** or *null* vector, denoted by $\vec{0}$ or **0**, which has a magnitude of 0 and no specific direction. Subtracting vectors is similar to subtraction with integers. To find **p** − **q**, add the opposite of **q** to **p**. That is, $\boxed{\mathbf{p} - \mathbf{q} = \mathbf{p} + (-\mathbf{q})}$.



$\mathbf{a} + (-\mathbf{a}) = \mathbf{0}$

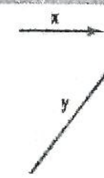
A vector can also be multiplied by a scalar.

KeyConcept Multiplying Vectors by a Scalar

If a vector **v** is multiplied by a real number scalar k, the scalar multiple $k\mathbf{v}$ has a magnitude of $|k|\ |\mathbf{v}|$. Its direction is determined by the sign of k.

- If $k > 0$, $k\mathbf{v}$ has the same direction as **v**.
- If $k < 0$, $k\mathbf{v}$ has the opposite direction as **v**.

Example 4 Operations with Vectors



Draw a vector diagram of $3\mathbf{x} - \frac{3}{4}\mathbf{y}$.

Rewrite the expression as the addition of two vectors: $3\mathbf{x} - \frac{3}{4}\mathbf{y} = 3\mathbf{x} + \left(-\frac{3}{4}\mathbf{y}\right)$. To represent $3\mathbf{x}$, draw a vector 3 times as long as $\mathbf{x}$ in the same direction as $\mathbf{x}$ (Figure 8.1.1). To represent $-\frac{3}{4}\mathbf{y}$, draw a vector $\frac{3}{4}$ the length of $\mathbf{y}$ in the opposite direction from $\mathbf{y}$ (Figure 8.1.2). Then use the triangle method to draw the resultant vector (Figure 8.1.3).

Figure 8.1.1

Figure 8.1.2

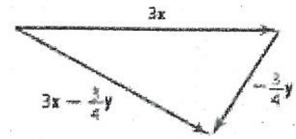
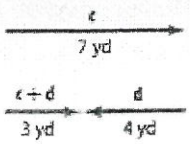


Figure 8.1.3

StudyTip

Parallel Vectors with Opposite Directions To add two parallel vectors with *opposite directions*, find the absolute value of the difference in their magnitudes. The resultant has the same direction as the vector with the greater magnitude.



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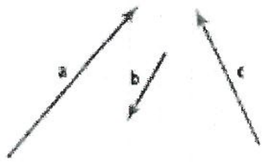
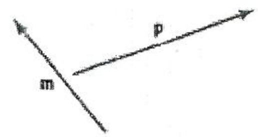
Draw a vector diagram of each expression.

* **4A.** $\mathbf{a} - \mathbf{c} + 2\mathbf{b}$

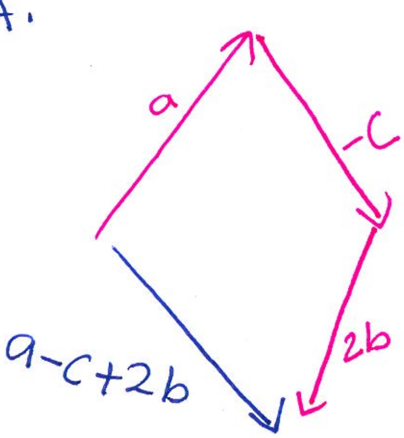
4B. $\mathbf{m} - \frac{1}{4}\mathbf{p}$



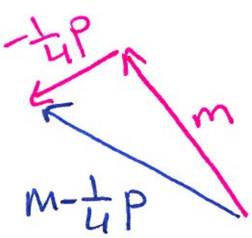


Extra credit due 5/6/15 sharp

4A.



4B.



2 Vector Applications

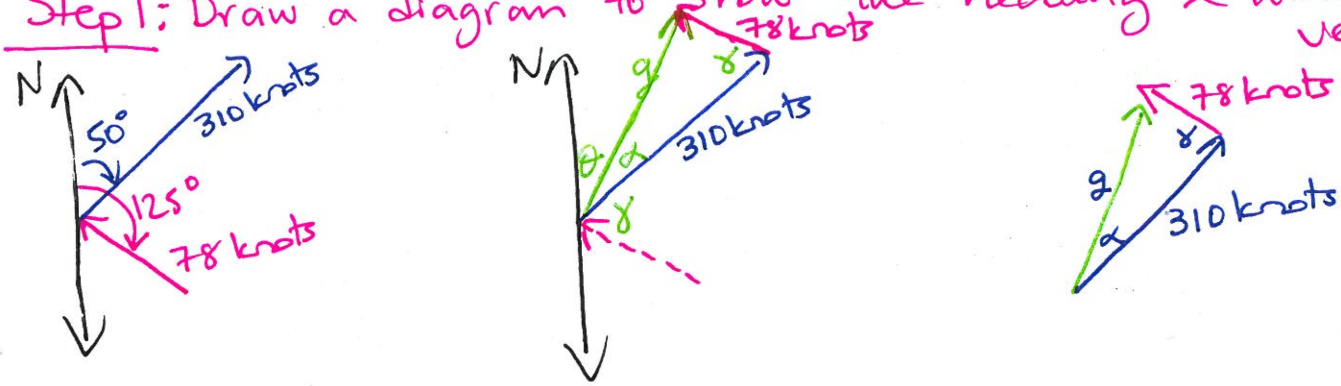
Vector Applications Vector addition and trigonometry can be used to solve vector problems involving triangles which are often oblique.

In navigation, a *heading* is the direction in which a vessel, such as an airplane or boat, is steered to overcome other forces, such as wind or current. The *relative velocity* of the vessel is the resultant when the heading velocity and other forces are combined.

Real-World Example 5 Use Vectors to Solve Navigation Problems

AVIATION An airplane is flying with an airspeed of 310 knots on a heading of 050°. If a 78-knot wind is blowing from a true heading of 125°, determine the speed and direction of the plane relative to the ground.

Step 1: Draw a diagram to show the heading & wind velocities



Extra credit due. 9/6 sharp

Step 2: Use the Law of Cosines to find $|g|$, the plane's speed relative to the ground.

$$\boxed{c^2 = a^2 + b^2 - 2ab\cos\gamma}$$ Law of Cosines

$$|g|^2 = 78^2 + 310^2 - 2(78)(310)\cos 75°$$

$$|g| = \sqrt{78^2 + 310^2 - 2(78)(310)\cos 75°}$$

$|g| \approx 299.4$

The ground speed of the plane is about 299.4 knots

Step 3: The heading of the resultant g is represented by angle θ, To calculate θ, find α using Law of Sines first

$$\frac{\sin\alpha}{a} = \frac{\sin\gamma}{c}$$

$$\frac{\sin\alpha}{78} = \frac{\sin 75°}{299.4}$$

$$\sin\alpha = \frac{78 \cdot \sin 75°}{299.4}$$

$$\alpha = \sin^{-1}\left(\frac{78 \cdot \sin 75°}{299.4}\right) \rightarrow \alpha \approx 14.6°$$

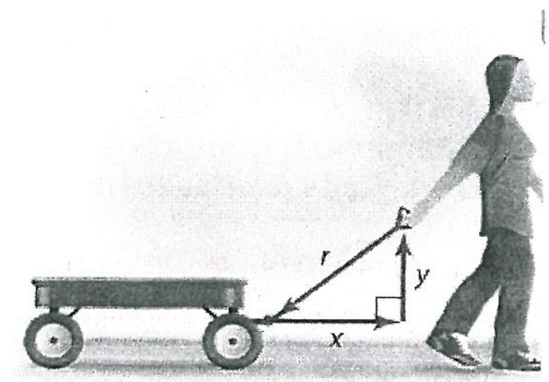
$$\theta = 50° - \alpha$$

$$\theta = 50° - 14.6°$$

$$\theta = 35.4°$$

The speed of the plane relative to the ground is about 299.4 knots at about 035°

Two or more vectors with a sum that is a vector **r** are called **components** of **r**. While components can have any direction, it is often useful to express or *resolve* a vector into two perpendicular components. The **rectangular components** of a vector are horizontal and vertical.

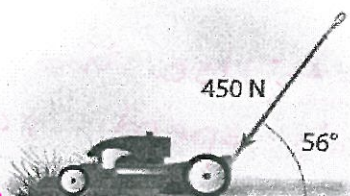


In the diagram, the force **r** exerted to pull the wagon can be thought of as the sum of a horizontal component force **x** that moves the wagon forward and a vertical component force **y** that pulls the wagon upward.

* Extra-credit Due 5/6 Thurs

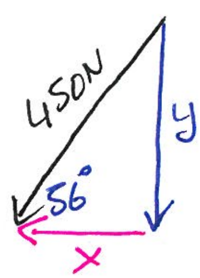
Real-World Example 6 Resolve a Force into Rectangular Components

LAWN CARE Heather is pushing the handle of a lawn mower with a force of 450 newtons at an angle of 56° with the ground.



a. Draw a diagram that shows the resolution of the force that Heather exerts into its rectangular components.

Heather's push can be resolved into horizontal push x forward and a vertical push y downward



b. Find the magnitudes of the horizontal and vertical components of the force.

Then, using the right triangle above,

$\cos 56° = \frac{|x|}{450}$ $\sin 56° = \frac{|y|}{450}$

$|x| = 450\cos 56°$ $|y| = 450 \sin 56°$

$|x| \approx 252$ $|y| \approx 373$