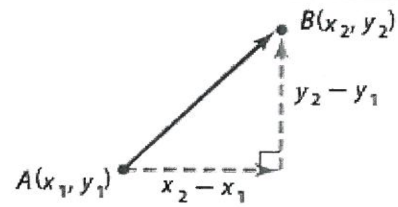
NAME ______________________ DATE ______________________ PERIOD __________

8.2 Vectors in the Coordinate Plane

KeyConcept Component Form of a Vector

The component form of a vector $\overrightarrow{AB}$ with initial point $A(x_1, y_1)$ and terminal point $B(x_2, y_2)$ is given by

$$\boxed{\langle x_2 - x_1, y_2 - y_1 \rangle.}$$



Express Vectors in Component Form:

Examples: *Find the component form of $\overrightarrow{AB}$ with the given initial and terminal points.*

1. $A(-2,-7), B(6,1)$

$\overrightarrow{AB} = \langle 6-(-2), 1-(-7) \rangle$

$\boxed{\overrightarrow{AB} = \langle 8, 8 \rangle}$

2. $A(0,8), B(-9,-3)$

$\overrightarrow{AB} = \langle -9-0, -3-8 \rangle$

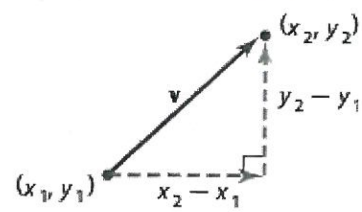
$\boxed{\overrightarrow{AB} = \langle -9, -11 \rangle}$

KeyConcept Magnitude of a Vector in the Coordinate Plane

If $\mathbf{v}$ is a vector with initial point (x_1, y_1) and terminal point (x_2, y_2), then the magnitude of $\mathbf{v}$ is given by

$$\boxed{|\mathbf{v}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.}$$

If $\mathbf{v}$ has a component form of $\langle a, b \rangle$, then $\boxed{|\mathbf{v}| = \sqrt{a^2 + b^2}.}$



Find the Magnitude of a Vector:

Examples: *Find the magnitude of $\overrightarrow{AB}$ with the given initial and terminal points.*

1. $A(-2,-7), B(6,1)$

$|\overrightarrow{AB}| = \sqrt{(6-(-2))^2 + (1-(-7))^2}$

$|\overrightarrow{AB}| = \sqrt{8^2 + 8^2}$

$= \sqrt{128}$

$= \boxed{11.31}$

2. $A(0,8), B(-9,-3)$

$|\overrightarrow{AB}| = \sqrt{(-9-0)^2 (-3-8)^2}$

$= \sqrt{(-9)^2 + (-11)^2}$

$= \sqrt{202}$

$= \boxed{14.21}$

KeyConcept Vector Operations

If $\mathbf{a} = \langle a_1, a_2\rangle$ and $\mathbf{b} = \langle b_1, b_2\rangle$ are vectors and k is a scalar, then the following are true.

Vector Addition	$\boxed{\mathbf{a} + \mathbf{b} = \langle a_1 + b_1, a_2 + b_2\rangle}$
Vector Subtraction	$\boxed{\mathbf{a} - \mathbf{b} = \langle a_1 - b_1, a_2 - b_2\rangle}$
Scalar Multiplication	$\boxed{k\mathbf{a} = \langle ka_1, ka_2\rangle}$

Operations with Vectors:

Examples: *Find each of the following for* $w = \langle -4, 1\rangle$, $y = \langle 2, 5\rangle$, $z = \langle -3, 0\rangle$.

1. $\mathbf{w} + \mathbf{y}$

$\langle -4, 1\rangle + \langle 2, 5\rangle$

$\langle -4+2, 1+5\rangle$

$\boxed{\langle -2, 6\rangle}$

2. $\mathbf{z} - 2\mathbf{y}$

$\langle -3, 0\rangle - 2\langle 2, 5\rangle$

$\langle -3, 0\rangle + \langle -4, -10\rangle$

$\langle -3+(-4), 0+(-10)\rangle$

$\boxed{\langle -7, -10\rangle}$

3. $2\mathbf{w} + 4\mathbf{y} - \mathbf{z}$

$2\langle -4, 1\rangle + 4\langle 2, 5\rangle - \langle -3, 0\rangle$

$\langle -8, 2\rangle + \langle 8, 20\rangle + \langle 3, 0\rangle$

$\langle 0, 22\rangle + \langle 3, 0\rangle$

$\boxed{\langle 3, 22\rangle}$

Unit Vectors: A vector that has a magnitude of 1 unit. It is sometimes useful to describe a nonzero vector $\mathbf{v}$ as a scalar multiple of a unit vector $\mathbf{u}$ with the same direction as $\mathbf{v}$. To find $\mathbf{u}$, divide $\mathbf{v}$ by its magnitude $|\mathbf{v}|$. $\quad \mathbf{u} = \frac{\mathbf{v}}{|\mathbf{v}|}$ or $\frac{1}{|\mathbf{v}|}\mathbf{v}$

Examples: *Find a unit vector with the same direction as the given vector.*

1. $\mathbf{v} = \langle -2, 3\rangle$

$|\vec{v}| = \sqrt{a^2 + b^2}$

$= \sqrt{(-2)^2 + 3^2}$

$= \sqrt{4 + 9}$

$= \sqrt{13}$

$\vec{u} = \frac{\vec{v}}{|\vec{v}|}$

$\vec{u} = \frac{\langle -2, 3\rangle}{\sqrt{13}}$

$\vec{u} = \left\langle \frac{-2}{\sqrt{13}}, \frac{3}{\sqrt{13}} \right\rangle$

$\vec{u} = \left\langle \frac{-2}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}}, \frac{3}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} \right\rangle$

$\boxed{\vec{u} = \left\langle \frac{-2\sqrt{13}}{13}, \frac{3\sqrt{13}}{13} \right\rangle}$

2. $\mathbf{v} = \langle -4, -8 \rangle$

$|\vec{v}| = \sqrt{(-4)^2 + (-8)^2}$

$= \sqrt{16 + 64}$

$= \sqrt{80}$

$\vec{u} = \frac{\vec{v}}{|\vec{v}|}$

$\vec{u} = \frac{\langle -4, -8 \rangle}{\sqrt{80}}$

$\vec{u} = \langle -\frac{4}{\sqrt{80}}, \frac{-8}{\sqrt{80}} \rangle$

$\vec{u} = \langle -\frac{4}{\sqrt{80}} \frac{\sqrt{80}}{\sqrt{80}}, \frac{-8}{\sqrt{80}} \frac{\sqrt{80}}{\sqrt{80}} \rangle$

$\vec{u} = \langle \frac{-4\sqrt{80}}{80}, -\frac{8\sqrt{80}}{80} \rangle$

$\sqrt{80} = 4\sqrt{5}$

$\vec{u} = \langle \frac{-4\sqrt{5}}{20}, \frac{-8\sqrt{5}}{20} \rangle$

$\boxed{\vec{u} = \langle -\frac{\sqrt{5}}{5}, \frac{-2\sqrt{5}}{5} \rangle}$

Linear Combinations of Unit Vectors: The sum of **ai+bj** is called a linear combination of the vectors I and j. Note that **i = <1, 0> and j = <0, 1> <a, b> = ai + bj**

Examples: *Let $\overrightarrow{DE}$ be the vector with the given initial and terminal points. Write $\overrightarrow{DE}$ as a linear combination of* **i** *and* **j**.

1. $D(-2, 3), E(4, 5)$

a. Find component form

$\overrightarrow{DE} = \langle 4-(-2), 5-3 \rangle = \langle 6, 2 \rangle$

b. Rewrite the vector as linear combination.

$\overrightarrow{DE} = \langle 6, 2 \rangle$

$\boxed{\overrightarrow{DE} = 6i + 2j}$

2. $D(-3, -8), E(-7, 1)$

$\overrightarrow{DE} = \langle -7-(-3), 1-(-8) \rangle$

$\overrightarrow{DE} = \langle -4, 9 \rangle$

$\boxed{\overrightarrow{DE} = -4i + 9j}$

Direction Angle: A way to specify the direction of vector $\mathbf{v} = \langle a, b \rangle$ is to state the direction angle θ that $\mathbf{v}$ makes with the positive x-axis.

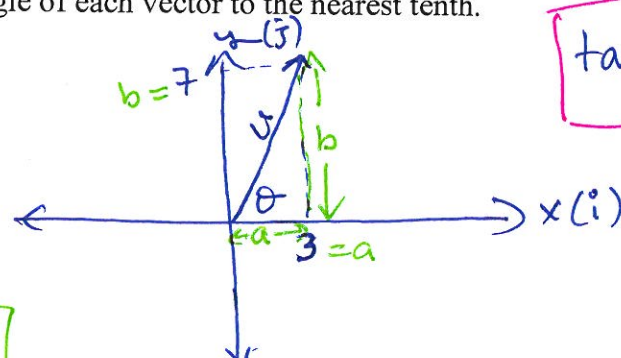
$$** \boxed{\mathbf{v} = \langle a, b \rangle = \langle |\mathbf{v}| \cos\theta, |\mathbf{v}| \sin\theta \rangle = |\mathbf{v}|(\cos\theta)\mathbf{i} + |\mathbf{v}|(\sin\theta)\mathbf{j}}$$

$\boxed{V = \langle a, b \rangle}$

$\boxed{\tan\theta = \frac{|V|\sin\theta}{|V|\cos\theta} = \frac{b}{a}}$

Examples: Find the direction of each angle of each vector to the nearest tenth.

1. $\mathbf{p} = 3\mathbf{i} + 7\mathbf{j}$ (a, b)



$\tan\theta = \frac{b}{a}$

$\tan\theta = \frac{7}{3}$

$\theta = \tan^{-1}\left(\frac{7}{3}\right) = \boxed{66.8^\circ}$

2. $\mathbf{r} = \langle 4, -5 \rangle$

$\tan\theta = \frac{-5}{4}$

$\theta = \tan^{-1}\left(\frac{-5}{4}\right)$

$\boxed{\theta = -51.3^\circ}$

Component Form Examples: *Find the component form of the vector* $\mathbf{v}$ *with the given magnitude and direction angle.*

1. $|\mathbf{v}| = 10, \theta = 120^\circ$

$V = \langle |V|\cos\theta, |V|\sin\theta \rangle$

$= \langle 10\cos 120^\circ, 10\sin 120^\circ \rangle$

$= \langle 10(-\frac{1}{2}), 10(\frac{\sqrt{3}}{2}) \rangle$

$\boxed{= \langle -5, 5\sqrt{3} \rangle}$

2. $|\mathbf{v}| = 8, \theta = 45^\circ$

$V = \langle 8\cos 45^\circ, 8\sin 45^\circ \rangle$

$= \langle 8(\frac{\sqrt{2}}{2}), 8(\frac{\sqrt{2}}{2}) \rangle$

$\boxed{= \langle 4\sqrt{2}, 4\sqrt{2} \rangle}$