Chapter 5 Test Review (Make sure that you are able to solve all the questions without looking at the solutions before the test)

KeyConcept Reciprocal and Quotient Identities

Reciprocal Identities			Quotient Identities
$\sin\theta = \frac{1}{\csc\theta}$	$\cos\theta = \frac{1}{\sec\theta}$	$\tan\theta = \frac{1}{\cot\theta}$	$\tan\theta = \frac{\sin\theta}{\cos\theta}$
$\csc\theta = \frac{1}{\sin\theta}$	$\sec\theta = \frac{1}{\cos\theta}$	$\cot\theta = \frac{1}{\tan\theta}$	$\cot\theta = \frac{\cos\theta}{\sin\theta}$

ReadingMath
Powers of Trigonometric Functions $\sin^2\theta$ is read as *sine squared theta* and interpreted as the square of the quantity $\sin\theta$.

KeyConcept Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$ $\quad \tan^2\theta + 1 = \sec^2\theta$ $\quad \cot^2\theta + 1 = \csc^2\theta$

StudyTip
Writing Cofunction Identities Each of the cofunction identities can also be written in terms of degrees. For example, $\sin\theta = \cos(90° - \theta)$.

KeyConcept Cofunction Identities

$\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$	$\tan\theta = \cot\left(\frac{\pi}{2} - \theta\right)$	$\sec\theta = \csc\left(\frac{\pi}{2} - \theta\right)$
$\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$	$\cot\theta = \tan\left(\frac{\pi}{2} - \theta\right)$	$\csc\theta = \sec\left(\frac{\pi}{2} - \theta\right)$

KeyConcept Odd-Even Identities

$\sin(-\theta) = -\sin\theta$	$\cos(-\theta) = \cos\theta$	$\tan(-\theta) = -\tan\theta$
$\csc(-\theta) = -\csc\theta$	$\sec(-\theta) = \sec\theta$	$\cot(-\theta) = -\cot\theta$

Practice

1) If $\tan\alpha = \frac{2}{3}$, find $\cot\alpha$.

$\cot\alpha = \frac{1}{\tan\alpha} = \frac{1}{2/3} = 1 \times \frac{3}{2} \Rightarrow \boxed{\cot\alpha = \frac{3}{2}}$

2) If $\cot\alpha = -\frac{4}{3}$ and $\sin\alpha < 0$, find $\cos\alpha$ and $\csc\alpha$.

* Pythagorean Identity
$\cot^2\theta + 1 = \csc^2\theta$

$\cot^2\alpha + 1 = \csc^2\alpha$

$\left(-\frac{4}{3}\right)^2 + 1 = \csc^2\alpha$

* $\sin\alpha$ is negative

$\csc\alpha = \frac{1}{\sin\alpha}$, so $\csc\alpha$ is negative

$\frac{16}{9} + 1 = \csc^2\alpha$

$\sqrt{\frac{25}{9}} = \sqrt{\csc^2\alpha} \rightarrow \boxed{\csc\alpha = -\frac{5}{3}}$ ✓

$\sin\alpha = \frac{1}{\csc\alpha} = \frac{1}{-5/3} = \frac{-3}{5}$

$\cot\alpha = \frac{\cos\alpha}{\sin\alpha} \rightarrow \cos\alpha = \cot\alpha \times \sin\alpha$

$\cos\alpha = \left(-\frac{4}{3}\right)\left(-\frac{3}{5}\right)$

$\boxed{\cos\alpha = \frac{4}{5}}$ ✓

3) If $\sec\beta = -2$, find $\csc\left(\beta - \frac{\pi}{2}\right)$.

* Cofunction
$\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta$

$\csc\left(\beta - \frac{\pi}{2}\right) = \csc\left[-\left(\frac{\pi}{2} - \beta\right)\right]$

$= -\csc\left(\frac{\pi}{2} - \beta\right) = -\sec\beta = -(-2) = +2$

4) $\sin x + \cos x \cot x = \sin x + \cos x \cdot \dfrac{\cos x}{\sin x}$, write all terms as $\sin x$ and $\cos x$

$= \dfrac{\sin x}{1}\,(\sin x) + \dfrac{\cos^2 x}{\sin x}$, LCD

$\sin^2 x + \cos^2 x = 1$ Pythagorean Identity

$= \dfrac{\sin^2 x}{\sin x} + \dfrac{\cos^2 x}{\sin x}$

$= \dfrac{\sin^2 x + \cos^2 x}{\sin x} = \dfrac{1}{\sin x} = \csc x$

5) $(\sec x - \tan x)(\csc x + 1) = \left(\dfrac{1}{\cos x} - \dfrac{\sin x}{\cos x}\right)\left(\dfrac{1}{\sin x} + \dfrac{1}{1}(\sin x)\right)$

$= \left(\dfrac{1-\sin x}{\cos x}\right)\left(\dfrac{1+\sin x}{\sin x}\right)$

$= \dfrac{1-\sin^2 x}{\sin x \cos x}$

$= \dfrac{\cancel{\cos^2 x}}{\sin x \cancel{\cos x}} = \dfrac{\cos x}{\sin x} = \cot x$

6) $(\cot^2 x + 1)(\sec^2 x - 1) = \csc^2 x \cdot \tan^2 x$ Pythagorean Identities

$* \cot^2\theta + 1 = \csc^2\theta$

$* \tan^2\theta + 1 = \sec^2 x$

$\tan^2\theta = \sec^2 x - 1$

$= \dfrac{1}{\cancel{\sin^2 x}} \times \dfrac{\cancel{\sin^2 x}}{\cos^2 x}$

$= \dfrac{1}{\cos^2 x}$

$= \sec^2 x$

7) $1 + \dfrac{\tan^2 x}{1 + \sec x} = 1 + \dfrac{\tan^2 x}{\frac{1}{1}(\cos x) + \frac{1}{\cos x}} = 1 + \dfrac{\frac{\sin^2 x}{\cos^2 x}}{\frac{\cos x + 1}{\cos x}}$

$= 1 + \dfrac{\sin^2 x}{\cos^{\cancel{2}} x} \times \dfrac{\cancel{\cos x}}{(\cos x + 1)}$

$\cos x$

$= \dfrac{1}{1}\,\cos x(\cos x + 1) + \dfrac{\sin^2 x}{\cos x(\cos x + 1)}$

$= \dfrac{\cos x(\cos x + 1) + \sin^2 x}{\cos x(\cos x + 1)}$

$= \dfrac{\cos^2 x + \cos x + \sin^2 x}{\cos x(\cos x + 1)} = \dfrac{\cos^2 x + \sin^2 x + \cos x}{\cos x(\cos x + 1)}$

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$= \dfrac{1 + \cancel{\cos x}}{\cos x(\cancel{\cos x + 1})} = \dfrac{1}{\cos x}$

$= \sec x$

8) $\dfrac{\cos\left(\frac{\pi}{2}-x\right)}{\csc x} + \cos^2 x = \dfrac{\sin x}{\csc x} + \cos^2 x = \dfrac{\sin x}{\frac{1}{\sin x}} + \cos^2 x$

* Cofunction Identity

$\sin x = \cos\left(\frac{\pi}{2}-x\right)$

$= \sin x \cdot \dfrac{\sin x}{1} + \cos^2 x$

$= \sin^2 x + \cos^2 x$

$= 1$

9) $\sec\theta - \cos\theta = \sin\theta\tan\theta$

Left-side:

$\sec\theta - \cos\theta = \dfrac{1}{\cos\theta} - \dfrac{\cos\theta}{1}\,(\cos\theta)$

$= \dfrac{1}{\cos\theta} - \dfrac{\cos^2\theta}{\cos\theta}$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$= \dfrac{1-\cos^2\theta}{\cos\theta}$

$= \dfrac{\sin^2\theta}{\cos\theta}$

$= \sin\theta \times \dfrac{\sin\theta}{\cos\theta}$

$= \sin\theta \times \tan\theta$

$= \sin\theta\tan\theta$ ✓

Thus, $\sec\theta - \cos\theta = \sin\theta\tan\theta$

$\sin\theta\tan\theta = \sin\theta\tan\theta$ ✓