Name KEY Date ________ Period ________

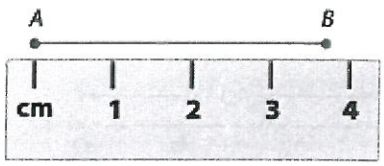
Lesson 1.2 Notes – Linear Measure

1 Measure Line Segments Unlike a line, a **line segment**, or *segment*, can be measured because it has two endpoints. A segment with endpoints A and B can be named as $\overline{AB}$ or $\overline{BA}$. The *measure* of $\overline{AB}$ is written as AB. The length or measure of a segment always includes a unit of measure, such as meter or inch.

Length in Metric Units

Find the length of $\overline{AB}$ using each ruler.

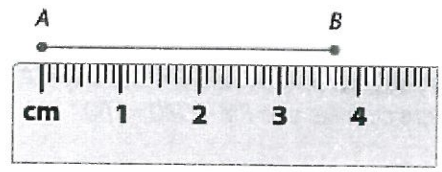
a.



The ruler is marked in centimeters. Point B is closer to the 4-centimeter mark than to 3 centimeters.

Thus, $\overline{AB}$ is about 4 centimeters long.

b.



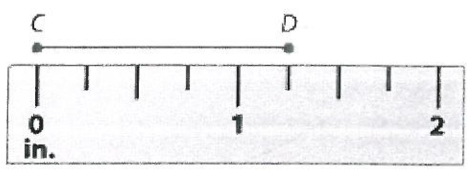
The long marks are centimeters, and the shorter marks are millimeters. There are 10 millimeters for each centimeter.

Thus, $\overline{AB}$ is about 3.7 centimeters long.

Length in Standard Units

Find the length of $\overline{CD}$ using each ruler.

a.

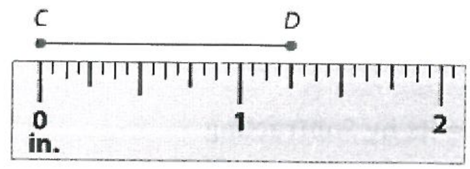


Each inch is divided into fourths.

Point D is closer to the $1\frac{1}{4}$-inch mark.

$\overline{CD}$ is about $1\frac{1}{4}$ inches long.

b.



Each inch is divided into sixteenths.

Point D is closer to the $1\frac{4}{16}$-inch mark.

$\overline{CD}$ is about $1\frac{4}{16}$ or $1\frac{1}{4}$ inches long.

Homework (Due 8/27) (Complete on this page)

1) Measure the length of a dollar bill both in metric units and standard units

15.6cm in metric units

6.25in in standard units

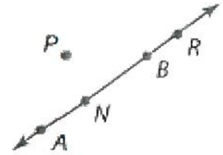
2) Measure the length of a new pencil both in metric units and standard units

19cm

7.5in

2 Calculate Measures

Recall that for any two real numbers a and b, there is a real number n that is *between* a and b such that $a < n < b$. This relationship also applies to points on a line and is called **betweenness of points**. In the figure, point N is between points A and B, but points R and P are not.



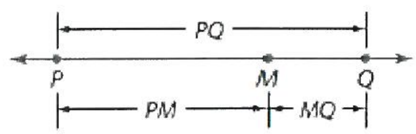
Measures are real numbers, so all arithmetic operations can be used with them. You know that the whole usually equals the sum of its parts. That is also true of line segments in geometry.

KeyConcept Betweenness of Points

Words

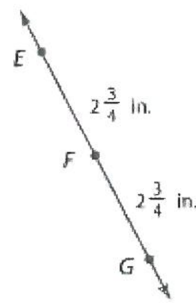
Point M is **between** points P and Q if and only if P, Q, and M are collinear and $PM + MQ = PQ$.

Model



Find Measurements by Adding

Find EG. Assume that the figure is not drawn to scale.



EG is the measure of $\overline{EG}$. Point F is between E and G. Find EG by adding EF and FG.

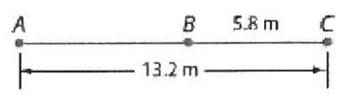
$EF + FG = EG$ Betweenness of points

$2\frac{3}{4} + 2\frac{3}{4} = EG$ Substitution

$5\frac{1}{2}$ in. $= EG$ Add.

Find Measurements by Subtracting

Find AB. Assume that the figure is not drawn to scale.



Point B is between A and C.

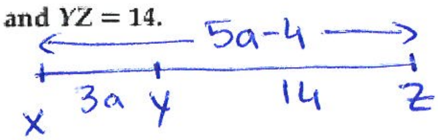
$AC = AB + BC$

$13.2 = AB + 5.8$

$13.2 - 5.8 = AB \rightarrow \boxed{AB = 7.4m}$

Find Measurements by Adding

ALGEBRA Find the value of a and XY if Y is between X and Z, $XY = 3a$, $XZ = 5a - 4$, and $YZ = 14$.



$XZ = XY + YZ$

$5a - 4 = 3a + 14$

$5a - 3a - 4 = 14$

$\frac{2a}{2} = \frac{18}{2}$ $\boxed{a = 9}$

Find XY

$XY = 3a$

$XY = 3 \times 9 = \boxed{27}$

Write and Solve Equations to Find Measurements

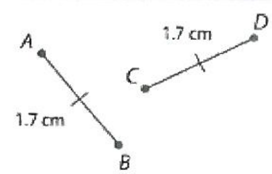
Congruent Segments

Segments that have the same measure are called **congruent segments**.

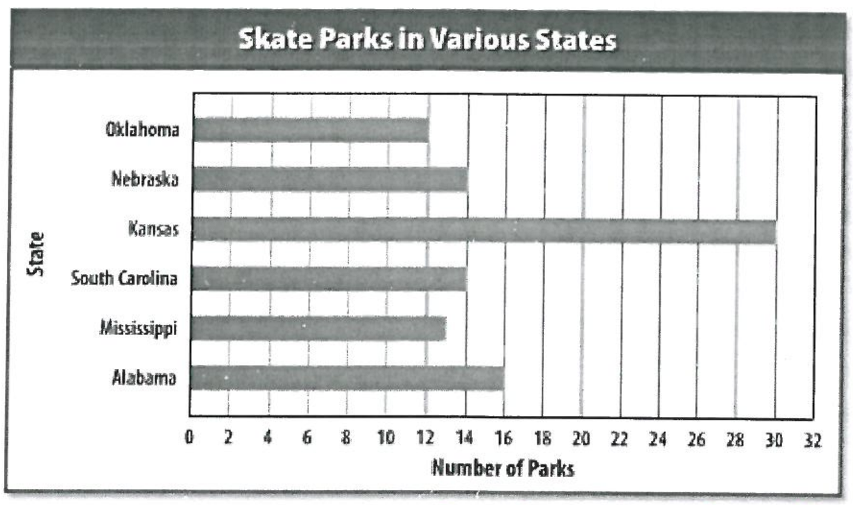
> **WatchOut!**
> Equal vs. Congruent Lengths are equal and segments are congruent. It is correct to say that $AB = CD$ and $\overline{AB} \cong \overline{CD}$. However, it is *not* correct to say that $\overline{AB} = \overline{CD}$ or that $AB \cong CD$.

KeyConcept Congruent Segments

Words	Congruent segments have the same measure.
Symbols	$\cong$ is read *is congruent to*. Red slashes on the figure also indicate congruence.
Example	$\overline{AB} \cong \overline{CD}$



SKATE PARKS **In the graph, suppose a segment was drawn along the top of each bar. Which states would have segments that are congruent? Explain.**



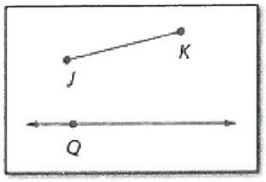
Since Nebraska and South Carolina are represented with the equal length segment bars, they are congruent

Drawings of geometric figures are created using measuring tools such as a ruler and protractor. **Constructions** are methods of creating these figures without the benefit of measuring tools. Generally, only a pencil, straightedge, and compass are used in constructions. *Sketches* are created without the use of any of these tools.

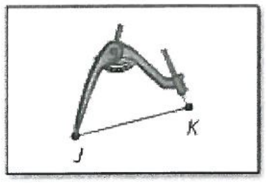
You can construct a segment that is congruent to a given segment.

Construction Copy a Segment

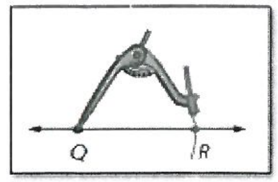
Step 1 Draw a segment $\overline{JK}$. Elsewhere on your paper, draw a line and a point on the line. Label the point Q.



Step 2 Place the compass at point J and adjust the compass setting so that the pencil is at point K.



Step 3 Using that setting, place the compass point at Q and draw an arc that intersects the line. Label the point of intersection R. $\overline{JK} \cong \overline{QR}$



Show your construction below.

Lesson 1.3 Notes – Distance and Midpoints

1 Distance Between Two Points The **distance** between two points is the length of the segment with those points as its endpoints. The coordinates of the points can be used to find this length. Because $\overline{PQ}$ is the same as $\overline{QP}$, the order in which you name the endpoints is not important when calculating distance.

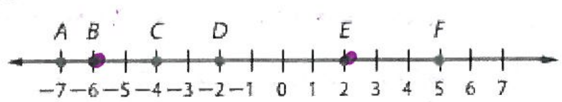
KeyConcept Distance Formula (on Number Line)

Words The distance between two points is the absolute value of the difference between their coordinates.

Symbols If P has coordinate x_1 and Q has coordinate x_2, $PQ = |x_2 - x_1|$ or $|x_1 - x_2|$.

Find the Distance on a Number Line

Use the number line to find BE.



X_1 X_2

$BE = |X_2 - X_1|$

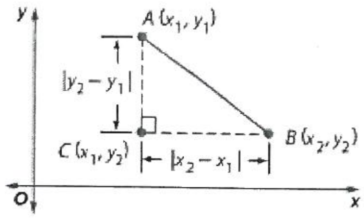
$BE = |2 - (-6)|$

$BE = |2 + 6| = \underline{\underline{8}}$

StudyTip

Pythagorean Theorem Recall that the Pythagorean Theorem is often expressed as $a^2 + b^2 = c^2$, where a and b are the measures of the shorter sides (legs) of a right triangle, and c is the measure of the longest side (hypotenuse). You will prove and learn about other applications of the Pythagorean Theorem in Lesson 8-2.

To find the distance between two points A and B in the coordinate plane, you can form a right triangle with $\overline{AB}$ as its hypotenuse and point C as its vertex as shown. Then use the Pythagorean Theorem to find AB.



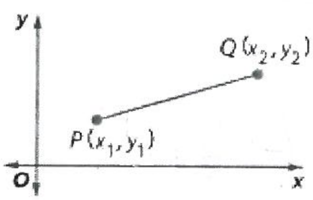
$(CB)^2 + (AC)^2 = (AB)^2$	Pythagorean Theorem
$(\lvert x_2 - x_1 \rvert)^2 + (\lvert y_2 - y_1 \rvert)^2 = (AB)^2$	$CB = \lvert x_2 - x_1 \rvert$, $AC = \lvert y_2 - y_1 \rvert$
$(x_2 - x_1)^2 + (y_2 - y_1)^2 = (AB)^2$	The square of a number is always positive.
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = AB$	Take the positive square root of each side.

This gives us a Distance Formula for points in the coordinate plane. Because this formula involves taking the square root of a real number, distances can be irrational. Recall that an **irrational number** is a number that cannot be expressed as a terminating or repeating decimal.

KeyConcept Distance Formula (in Coordinate Plane)

If P has coordinates (x_1, y_1) and Q has coordinates (x_2, y_2), then

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$



The order of the x- and y-coordinates in each set of parentheses is not important.

Find the Distance on a Coordinate Plane

Find the distance between $C(-4, -6)$ and $D(5, -1)$. Identify x_1, y_1 and x_2, y_2

$CD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

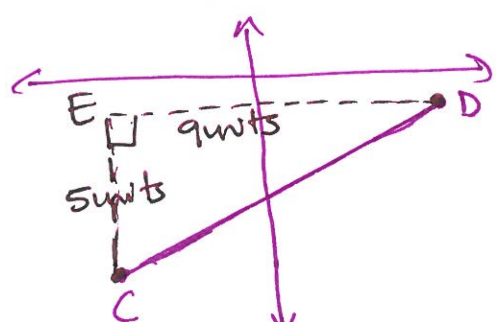
$CD = \sqrt{(5-(-4))^2 + (-1-(-6))^2} = \sqrt{9^2 + 5^2} = \sqrt{106} \approx 10.3$ units

Check your answer by using Pythagorean Theorem

$CD^2 \overset{?}{=} EC^2 + ED^2$

$CD^2 = 5^2 + 9^2$

$CD^2 = \sqrt{106} \cong 10.3$ units

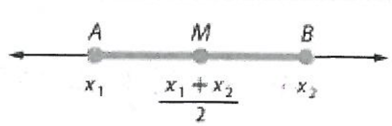


2 Midpoint of a Segment The **midpoint** of a segment is the point halfway between the endpoints of the segment. If X is the midpoint of $\overline{AB}$, then $AX = XB$ and $\overline{AX} \cong \overline{XB}$. You can find the midpoint of a segment on a number line by finding the *mean*, or the average, of the coordinates of its endpoints.

KeyConcept Midpoint Formula (on Number Line)

If $\overline{AB}$ has endpoints at x_1 and x_2 on a number line, then the midpoint M of $\overline{AB}$ has coordinate

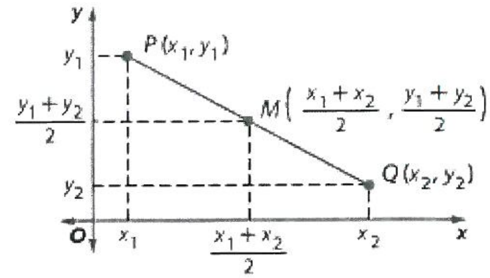
$$\frac{x_1 + x_2}{2}.$$



KeyConcept Midpoint Formula (in Coordinate Plane)

If $\overline{PQ}$ has endpoints at $P(x_1, y_1)$ and $Q(x_2, y_2)$ in the coordinate plane, then the midpoint M of $\overline{PQ}$ has coordinates

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$



When finding the midpoint of a segment, the order of the coordinates of the endpoints is not important.

Find Midpoint in Coordinate Plane

Find the coordinates of M, the midpoint of $\overline{ST}$, for $S(-6, 3)$ and $T(1, 0)$.

$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ Midpoint Formula

$M = \left(\frac{-6+1}{2}, \frac{3+0}{2}\right)$ $(x_1, y_1) = S(-6, 3)$, $(x_2, y_2) = T(1, 0)$

$= \left(-\frac{5}{2}, \frac{3}{2}\right)$ or $M\left(-2\frac{1}{2}, 1\frac{1}{2}\right)$ Simplify

Find the Coordinates of an Endpoint

Find the coordinates of J if $K(-1, 2)$ is the midpoint of $\overline{JL}$ and L has coordinates $(3, -5)$.

Step 1 Let J be (x_1, y_1) and L be (x_2, y_2) in the Midpoint Formula.

$K\left(\frac{x_1 + 3}{2}, \frac{y_1 + (-5)}{2}\right) = K(-1, 2)$ $(x_2, y_2) = (3, -5)$

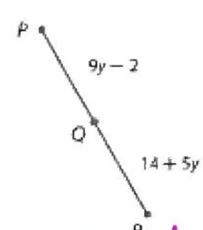
Step 2 Write two equations to find the coordinates of J.

$\frac{x_1 + 3}{2} = -1$	Midpoint Formula	$\frac{y_1 + (-5)}{2} = 2$	Midpoint Formula
$x_1 + 3 = -2$	Multiply each side by 2.	$y_1 - 5 = 4$	Multiply each side by 2.
$x_1 = -5$	Subtract 3 from each side.	$y_1 = 9$	Add 5 to each side.

The coordinates of J are $(-5, 9)$.

Use Algebra to Find Measures

ALGEBRA Find the measure of $\overline{PQ}$ if Q is the midpoint of $\overline{PR}$.



Understand You know that Q is the midpoint of $\overline{PR}$. You are asked to find the measure of $\overline{PQ}$.

Plan Because Q is the midpoint, you know that $PQ = QR$. Use this equation to find a value for y.

Solve $PQ = QR$ Definition of Midpoint

$9y - 2 = 14 + 5y$

$4y = 16$

$y = 4$ substitute

$PQ = 9y - 2 = 9 \times 4 - 2 = \boxed{34}$

Check $QR = 14 + 5y = 14 + 5 \times 4 = \boxed{34}$

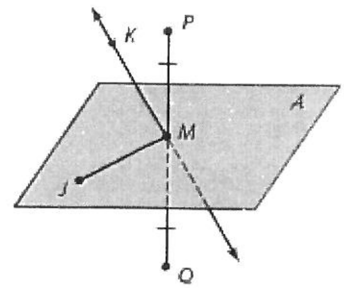
$PQ = QR$ – same length

Segment Bisectors

StudyTip

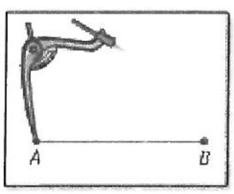
Segment Bisectors There can be an infinite number of bisectors and each must contain the midpoint of the segment.

Any segment, line, or plane that intersects a segment at its midpoint is called a **segment bisector**. In the figure at the right, M is the midpoint of $\overline{PQ}$. Plane $\mathcal{A}$, $\overline{MJ}$, $\overleftrightarrow{KM}$, and point M are all bisectors of $\overline{PQ}$. We say that they *bisect* $\overline{PQ}$.

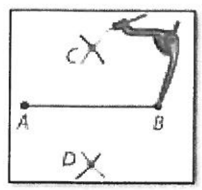


Construction Bisect a Segment

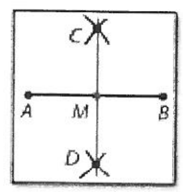
Step 1 Draw a segment and name it $\overline{AB}$. Place the compass at point A. Adjust the compass so that its width is greater than $\frac{1}{2}AB$. Draw arcs above and below $\overline{AB}$.



Step 2 Using the same compass setting, place the compass at point B and draw arcs above and below $\overline{AB}$ so that they intersect the two arcs previously drawn. Label the points of the intersection of the arcs as C and D.



Step 3 Use a straightedge to draw $\overline{CD}$. Label the point where it intersects $\overline{AB}$ as M. Point M is the midpoint of $\overline{AB}$, and $\overline{CD}$ is a bisector of $\overline{AB}$.



Show your construction below.

ANSWER ALL 4 QUESTIONS IN YOUR GEOMETRY NOTEBOOK! NO WORK = 0 GRADE!!!

KEEP THIS NOTES IN YOUR BINDER. IF YOU LOOSE IT = 0 GRADE!!!