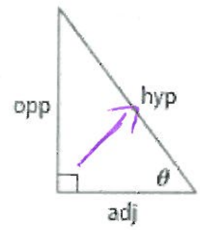
Pre-Calculus Notes on Right Triangle Trigonometry 1/22/2015

Objective: I can find values of trigonometric functions for acute angles of right triangles; I can solve right triangle problems.

KeyConcept Trigonometric Functions

Let θ be an acute angle in a right triangle and the abbreviations opp, adj, and hyp refer to the length of the side opposite θ, the length of the side adjacent to θ, and the length of the hypotenuse, respectively.



Then the six trigonometric functions of θ are defined as follows.

sine $(\theta) = \sin\theta = \frac{\text{opp}}{\text{hyp}}$ SOH

cosine $(\theta) = \cos\theta = \frac{\text{adj}}{\text{hyp}}$ CAH

tangent $(\theta) = \tan\theta = \frac{\text{opp}}{\text{adj}}$ TOA

cosecant $(\theta) = \csc\theta = \frac{\text{hyp}}{\text{opp}}$ CHO

secant $(\theta) = \sec\theta = \frac{\text{hyp}}{\text{adj}}$ SHA

cotangent $(\theta) = \cot\theta = \frac{\text{adj}}{\text{opp}}$ CAO

! Hypotenuse is always across from the right angle.

! Opposite side and adjacent side changes depending on where θ is located.

The cosecant, secant, and cotangent functions are called **reciprocal functions** since their ratios are reciprocals of the sine, cosine, and tangent ratios, respectively.

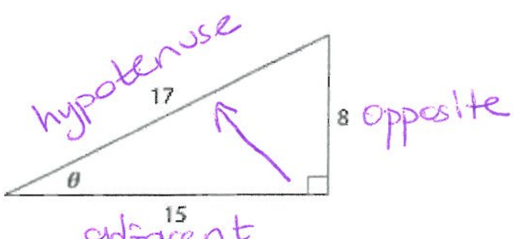
$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

You can also derive the following relationships from the definitions of the sine, cosine, tangent, and cotangent functions.

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \quad \text{and} \quad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Examples: Find the exact values of six geometric functions of θ.

1.



$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{8}{17}$

$\cos\theta = \frac{\text{adj.}}{\text{hyp}} = \frac{15}{17}$

$\tan\theta = \frac{\text{opp}}{\text{adj.}} = \frac{8}{15}$

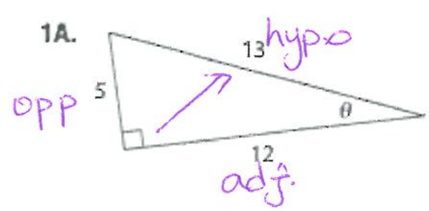
$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{17}{8}$

$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{17}{15}$

$\cot\theta = \frac{\text{adj.}}{\text{opp}} = \frac{15}{8}$

2.

1A.



$\sin\theta = \frac{5}{13}$

$\cos\theta = \frac{12}{13}$

$\tan\theta = \frac{5}{12}$

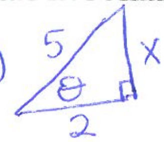
$\csc\theta = \frac{13}{5}$

$\sec\theta = \frac{13}{12}$

$\cot\theta = \frac{12}{5}$

3. If $\cos\theta = \frac{2}{5}$, find the exact values of the five remaining trigonometric functions for the acute angle θ.

1) $\cos\theta = \frac{adj.}{hyp} = \frac{2}{5}$

2) 

3) Use Pythagorean Thm to find 3rd side

$x^2 + 2^2 = 5^2$

$x^2 + 4 = 25$

$\sqrt{x^2} = \sqrt{21}$

$x = \sqrt{21}$

$\boxed{\sin\theta = \frac{\sqrt{21}}{5}}$

$\boxed{\tan\theta = \frac{\sqrt{21}}{2}}$

$\boxed{\sec\theta = \frac{5}{2}}$

$\csc\theta = \frac{5}{\sqrt{21}} = \frac{5}{\sqrt{21}} \cdot \frac{\sqrt{21}}{\sqrt{21}}$

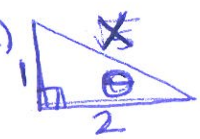
$\boxed{\csc\theta = \frac{5\sqrt{21}}{21}}$

$\cot\theta = \frac{2}{\sqrt{21}} = \frac{2}{\sqrt{21}} \cdot \frac{\sqrt{21}}{\sqrt{21}} = \boxed{\frac{2\sqrt{21}}{21}}$

*Make sure you rationalize the denominator if it has a radical

4. If $\tan\theta = \frac{1}{2}$, find the exact values of the five remaining trigonometric functions for the acute angle θ.

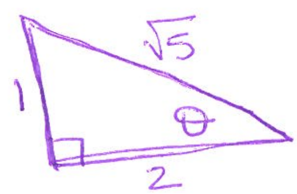
1) $\tan\theta = \frac{opp}{adj.} = \frac{1}{2}$

2) 

3) $1^2 + 2^2 = x^2$

$\sqrt{5} = \sqrt{x^2}$

$5 = x$



$\sin\theta = \frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$ //

$\cos\theta = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$ //

$\tan\theta = \frac{1}{2}$ //

$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{2/\sqrt{5}} \Rightarrow \sec\theta = \frac{\sqrt{5}}{2}$ //

$\cot\theta = \frac{2}{1} = 2$ //

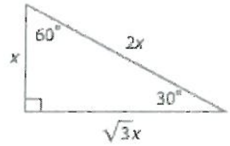
$\csc\theta = \frac{1}{\sin\theta}$

$\csc\theta = \frac{1}{1/\sqrt{5}}$

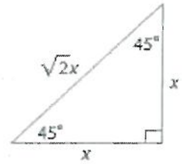
$\csc\theta = \sqrt{5}$ //

KeyConcept Trigonometric Values of Special Angles

30°-60°-90° Triangle



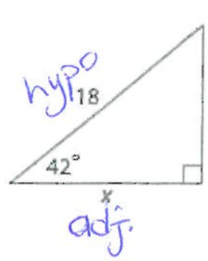
45°-45°-90° Triangle



θ	30°	45°	60°
sin θ	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cos θ	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tan θ	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$
csc θ	2	$\sqrt{2}$	$\frac{2\sqrt{3}}{3}$
sec θ	$\frac{2\sqrt{3}}{3}$	$\sqrt{2}$	2
cot θ	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$

How can we find a Missing Side Length or Angle Measure in a Right Triangle?

Example: Find the value of x. Round it to nearest tenth if necessary.



$\frac{adj}{hypo} = \cos 42°$

$\frac{x}{18} = \cos 42°$ multiply both sides by 18

$x = 18(\cos 42°)$

$\boxed{x = 13.4}$