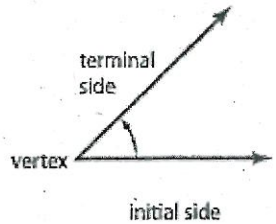
Name_______________________________________ Date____________________

4.2 Degrees and Radians

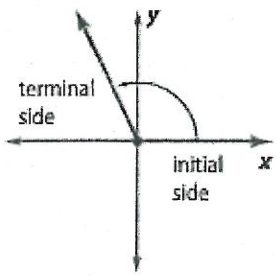
Obj: I can convert degrees to radians and vice versa.

Angles and Their Measures

Angle

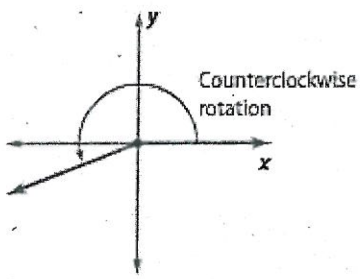


Angle in Standard Position

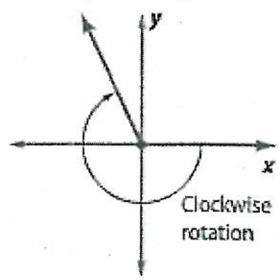


A positive angle is generated by a counterclockwise rotation and **a negative angle** by a clockwise rotation.

Positive Angle



Negative Angle

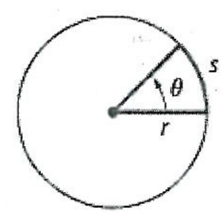


The most common angular unit of measure is the *degree* (°), which is equivalent to $\frac{1}{360}$ of a full rotation (counterclockwise) about the vertex.

Degrees and Radians: The radian measure of the central angle, angle AOB, is the number of radius units in the length of arc AB.

KeyConcept Radian Measure

Words	The measure θ in radians of a central angle of a circle is equal to the ratio of the length of the intercepted arc s to the radius r of the circle.
Symbols	$\boxed{\theta = \frac{s}{r}}$, where θ is measured in radians (rad)
Example	A central angle has a measure of 1 radian if it intercepts an arc with the same length as the radius of the circle.



$\theta = 1$ radian when $s = r$.

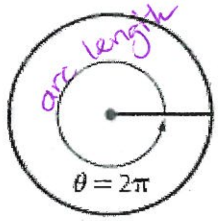
*** One revolution in radians is 2π and measured in degrees is 360.

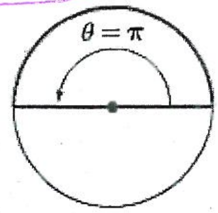
**** Conversions:**

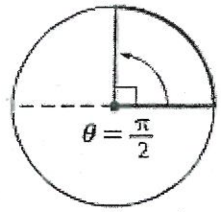
$\boxed{1 \text{ radian} = \frac{180}{\pi} \text{ degrees}}$ $\boxed{1 \text{ degree} = \frac{\pi}{180} \text{ radians}}$

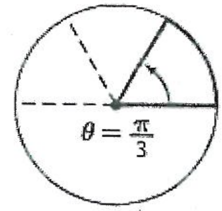
The central angle representing one full rotation counterclockwise about a vertex corresponds to an arc length equivalent to the circumference of the circle, $2\pi r$. From this, you can obtain the following radian measures.

Remember $\boxed{\theta = \frac{s}{r}}$









1 rotation = $\frac{2\pi r}{r}$ $= 2\pi$ rad

$\frac{1}{2}$ rotation = $\frac{1}{2} \cdot 2\pi$ $= \pi$ rad

$\frac{1}{4}$ rotation = $\frac{1}{4} \cdot 2\pi$ $= \frac{\pi}{2}$ rad

$\frac{1}{6}$ rotation = $\frac{1}{6} \cdot 2\pi$ $= \frac{\pi}{3}$ rad.

Examples: Write each degree measure in radians as a multiple of π and each radian measure in degrees.

1. $120° = 120°\left(\frac{\pi \text{ radians}}{180°}\right)$

   $= \frac{2\pi}{3}$ radians

   OR $120° = \frac{2\pi}{3}$

2. $-45° = -45°\left(\frac{\pi \text{ radians}}{180°}\right)$

   $-45° = -45\left(\frac{\pi}{180°}\right)$

   $-45° = -\frac{\pi}{4}$ radians

   $-45° = -\frac{\pi}{4}$

3. $\frac{5\pi}{6} = \frac{5\pi}{6}$ radians

   $\frac{5\pi}{6} = \frac{5\cancel{\pi}}{6}$ radians $\left(\frac{180°}{\cancel{\pi} \text{ radians}}\right)$

   $\frac{5\pi}{6} = 150°$

4. $-\frac{3\pi}{2} = -\frac{3\pi}{2}$ radians

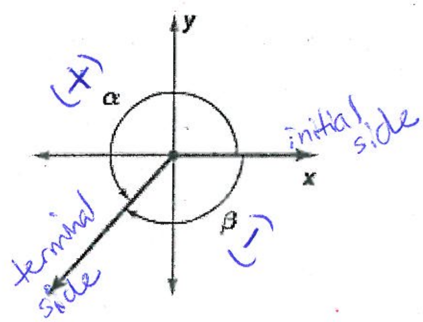
   $= -\frac{3\pi}{2}$ radians $\left(\frac{180°}{\pi \text{ radians}}\right)$

   $-\frac{3\pi}{2} = -270°$

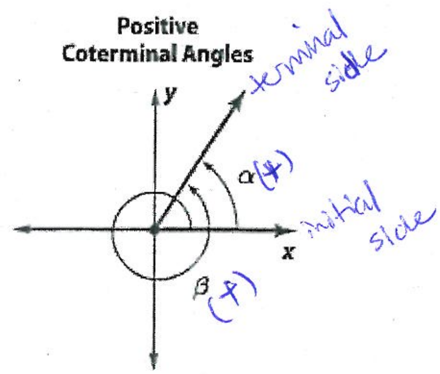
Coterminal Angles:

By defining angles in terms of their rotation about a vertex, two angles can have the same initial and terminal sides but different measures. Such angles are called **coterminal angles**. In the figures below, angles α and β are coterminal.

Positive and Negative Coterminal Angles



Positive Coterminal Angles



The two positive coterminal angles shown differ by one full rotation. A given angle has infinitely many coterminal angles found by adding or subtracting integer multiples of 360° or 2π radians.

KeyConcept Coterminal Angles

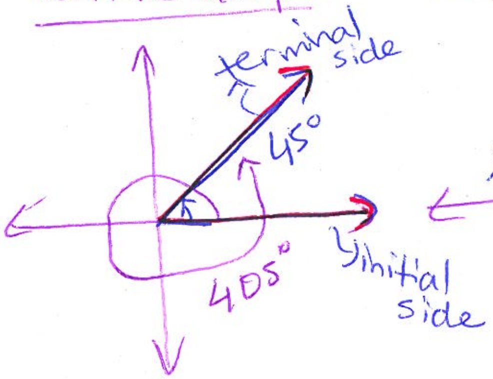
Degrees	Radians
If α is the degree measure of an angle, then all angles measuring $\alpha + 360n°$, where n is an integer, are coterminal with α.	If α is the radian measure of an angle, then all angles measuring $\alpha + 2n\pi$, where n is an integer, are coterminal with α.

Examples: Identify all angles that are coterminal with the given angle. Then find and draw one positive and one negative angle coterminal with the given angle.

1. 45°

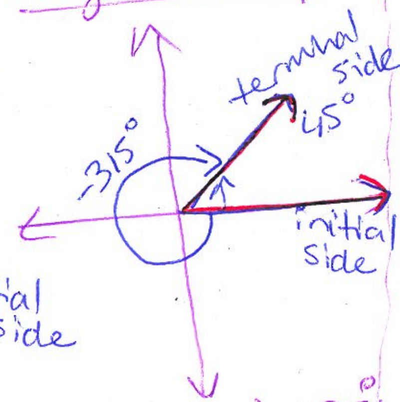
For any integer n, all coterminal angles $= 45^\circ + 360n$ ✱ remember

Positive Example



$45^\circ + 360(1) = 405^\circ$

Negative Example

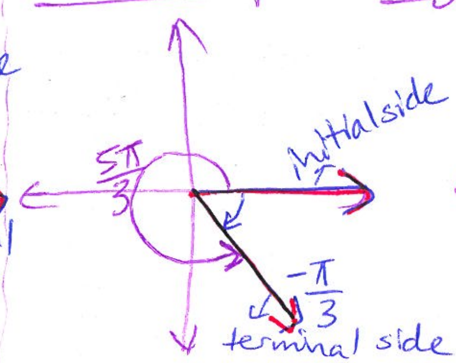


$45^\circ + 360(-1) = -315^\circ$

2. $-\frac{\pi}{3}$

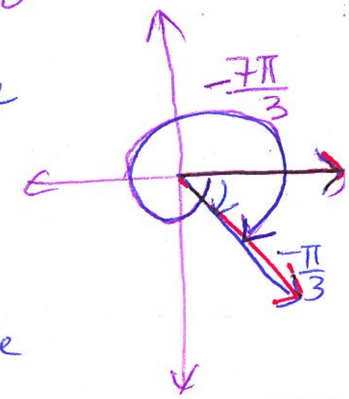
For any integer n, all coterminal angles $= -\frac{\pi}{3} + 2\pi n$ ✱ remember

Positive Example



$-\frac{\pi}{3} + 2\pi(1) = \frac{5\pi}{3}$

Negative Example



$-\frac{\pi}{3} + 2\pi(-1) = -\frac{7\pi}{3}$

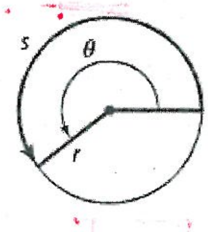
Applications with Angle Measure:

KeyConcept Arc Length

If θ is a central angle in a circle of radius r, then the length of the intercepted arc s is given by

$$\boxed{s = r\theta,}$$

where θ is measured in radians.



When θ is measured in degrees, you could also use the equation $\boxed{s = \frac{\pi r \theta}{180}}$, which already

*** incorporates the degree-radian conversion.

Examples: Find the length of the intercepted arc in each circle with the given central angle measure and radius. Round to the nearest tenth. Make sure to convert angles to radians.

1. $\frac{\pi}{4}$, $r = 5cm$, $\theta = \frac{\pi}{4}$

$s = r\theta$

$s = (5\text{ cm})\frac{\pi}{4}$

$s = \frac{5\pi}{4}\text{cm}$, $\pi = 3.14$

$s = \frac{5}{4} \times 3.14\text{ cm} = 3.9\text{cm}$

2. 60°, $r = 2\ in$

$60^\circ = 60^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{\pi}{3}$ ✱

$s = r\theta$

$s = 2in\left(\frac{\pi}{3}\right)$

$s = \frac{2\pi}{3}\text{in}$

$s = \frac{2 \times 3.14}{3} = 2.1in$

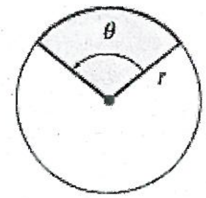
Area of a Sector: Recall from geometry that a ***sector*** of a circle is a region bounded by a central angle and its intercepted arc.

KeyConcept Area of a Sector

The area A of a sector of a circle with radius r and central angle θ is

$$A = \frac{1}{2}r^2\theta,$$

where θ is measured in radians.



Examples: Find the area of the sector of the circle. Make sure to convert angles to radians.

1. $\theta = \frac{7\pi}{8}$, $r = 3cm$

$A = \frac{1}{2}(3)^2\left(\frac{7\pi}{8}\right)$

$A = \frac{9}{2}(3)^2\left(\frac{7\pi}{8}\right)$

$A = \frac{63}{16}\pi$

$A = 12.4cm^2$

2. $\theta = 50°$, $r = 6m$

$50° = \frac{5\pi}{18}$

$A = \frac{1}{2}6^2\left(\frac{5\pi}{18}\right)$

$A = \frac{1}{2}36\left(\frac{5\pi}{18}\right)$

$A = 18\left(\frac{5\pi}{18}\right)$

$A = 5\pi = 5 \times 3.14$

$\boxed{A = 15.7m^2}$

Real – Life Examples:

The rate at which an object *moves along a circular path* is called its linear speed Units miles/hour

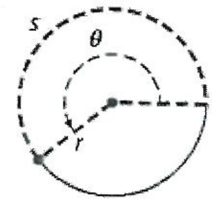
The rate at which the object *rotates about a fixed point* is called its angular speed Units revolutions per minute

KeyConcept Linear and Angular Speed

Suppose an object moves at a constant speed along a circular path of radius r.

If s is the arc length traveled by the object during time t, then the object's *linear speed* v is given by $\boxed{v = \frac{s}{t}}$.

If θ is the angle of rotation (in radians) through which the object moves during time t, then the *angular speed* ω of the object is given by $\boxed{\omega = \frac{\theta}{t}}$.



Examples:

1) Tires with a diameter of 30 in rotate at a rate of 140 revolutions per minute during one delivery. Find the angular speed of the tire in radians per minute.

$\theta = 140$ revolutions/min $\Rightarrow$ Complete circle is 2π radians. So,

$$\boxed{\frac{1 \text{ revolution}}{\text{min}} = 2\pi}$$

$\theta = 140 \cdot (2\pi)$

$= 280\pi$

$w = \frac{\theta}{t}$

$w = \frac{280\pi}{1 \text{ min}}$

$w = 280\pi = 280 \cdot (3.14)$

$w = 879.6$ rad/min

2) During next delivery, the tire turns at a constant rate of 2.5 revolutions per second. Find linear speed of the tire in miles per hour.

$\theta = 2.5$ revolutions/sec

$\boxed{\frac{1 \text{ revolution}}{\text{second}} = 2\pi \text{ in radians}}$

$\theta = 2.5(2\pi)$

$v = \frac{75\pi \cancel{in}}{1 \cancel{sec}} \times \frac{60 \cancel{sec}}{1 min} \times \frac{60 min}{1 hr} \times \frac{1 \cancel{ft}}{12 \cancel{in}} \times \frac{1 mile}{5280 \cancel{ft}}$

$\theta = 5\pi$

Then, Linear speed

$\boxed{v \approx 13.4 \text{ miles/hour}}$

$v = \frac{s}{t} = \frac{r\theta}{t} = \frac{15(2\pi) \text{ inc}}{1 \text{ sec}}$

$\boxed{v = 75\pi \text{ in/sec}}$ → we need to convert into mi/hr

Homework: Due Feb 9, 2015

Write each decimal degree measure in DMS form and each DMS measure in decimal degree form to the nearest thousandth.

1. 28.955

2. –57.3278

3. 32 28' 10"

4. –73 14' 35"

Write each degree measure in radians as a multiple of π and each radian measure in degrees.

5. 25°

6. 130°

7. $\frac{3\pi}{4}$

8. $\frac{5\pi}{3}$

Identify all angles that are coterminal with the given angle. Then find and draw one positive and one negative angle coterminal with the given angle.

9. 43°

10. $-\frac{7\pi}{4}$

Find the length of the intercepted arc with the given central angle measure in a circle of the given radius. Round to the nearest tenth.

11. 30°, $r = 8$ yd

12. $\frac{7\pi}{6}$, $r = 10$ in.

Find the area of each sector.

13. $\theta = \frac{\pi}{6}$, $r = 14$ in.

14. $\theta = \frac{7\pi}{4}$, $r = 4$ m

Find the rotation in revolutions per minute given the angular speed and the radius given the linear speed and the rate of rotation.

15. $\omega = \frac{4}{5}\pi$ rad/s

16. $V = 32$ m/s, 100 rev/min

17. On a game show, a contestant spins a wheel. The angular speed of the wheel was $\omega = \frac{\pi}{3}$ radians per second. If the wheel maintained this rate, what would be the rotation in revolutions per minute?