Name________________________ Section____________ Date____________

4.3 Trigonometric Functions on the Unit Circle

KeyConcept Trigonometric Functions of Any Angle

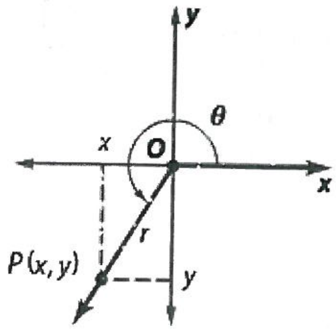
Let θ be any angle in standard position and point $P(x, y)$ be a point on the terminal side of θ. Let r represent the nonzero distance from P to the origin.

That is, let $\boxed{r = \sqrt{x^2 + y^2} \neq 0}$. Then the trigonometric functions of θ are as follows.

$\sin\theta = \frac{y}{r}$ $\qquad$ $\csc\theta = \frac{r}{y},\ y \neq 0$

$\cos\theta = \frac{x}{r}$ $\qquad$ $\sec\theta = \frac{r}{x},\ x \neq 0$

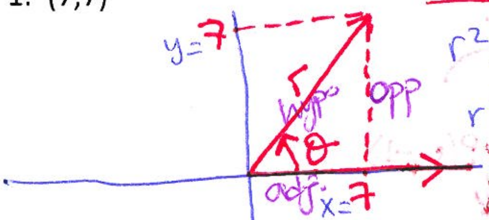
$\tan\theta = \frac{y}{x},\ x \neq 0$ $\qquad$ $\cot\theta = \frac{x}{y},\ y \neq 0$



Examples: The given points lie on the terminal side of an angle θ in standard position. Find the values of the six trigonometric functions of θ.

*rationalize denominator

1. (7,7)



Step1: Find r

$r^2 = x^2 + y^2$

$r^2 = 7^2 + 7^2$

$r = 2\sqrt{7}$

Step2: Find trig

① $\sin\theta = \frac{y}{r} = \frac{7}{2\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{7}}{2}$

② $\cos\theta = \frac{x}{r} = \frac{7}{2\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{7}}{2}$

③ $\tan\theta = \frac{y}{x} = \frac{7}{7} = 1$

④ $\cot\theta = \frac{x}{y} = 1$

⑤ $\sec\theta = \frac{2\sqrt{7}}{7}$

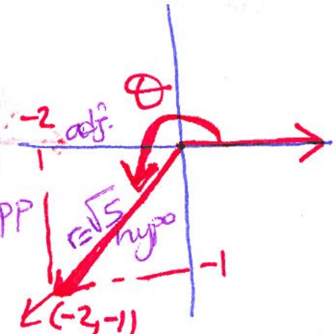
⑥ $\csc\theta = \frac{2\sqrt{7}}{7}$

2. (-2,-1)

$r^2 = x^2 + y^2$

$r^2 = 1^2 + 2^2$

$r^2 = 5 \rightarrow \boxed{r = \sqrt{5}}$



① $\sin\theta = \frac{-\sqrt{5}}{5}$

② $\cos\theta = \frac{-2}{\sqrt{5}} = \frac{-2\sqrt{5}}{5}$

③ $\tan\theta = \frac{y}{x} = \frac{-1}{-2} = \frac{1}{2}$

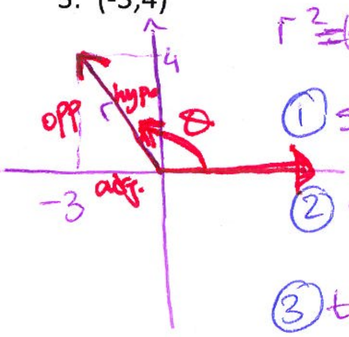
④ $\cot\theta = \frac{x}{y} = 2$

⑤ $\sec\theta = \frac{r}{x} = \frac{\sqrt{5}}{-2} = -\frac{\sqrt{5}}{2}$

⑥ $\csc\theta = \frac{r}{y}$

$\csc\theta = \frac{\sqrt{5}}{-1} = -\sqrt{5}$

3. (-3,4)



$r^2 = (-3)^2 + 4^2 = 9 + 16 = 25$

$\boxed{r = 5}$

① $\sin\theta = \frac{4}{5}$

② $\cos\theta = \frac{-3}{5}$

③ $\tan\theta = \frac{4}{-3} = -\frac{4}{3}$

④ $\cot\theta = \frac{-3}{4}$

⑤ $\sec\theta = \frac{5}{-3} = -\frac{5}{3}$

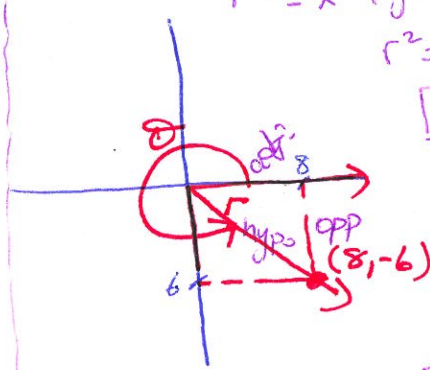
⑥ $\csc\theta = \frac{5}{4}$

4. (8,-6)

$r^2 = x^2 + y^2 = 8^2 + (-6)^2$

$r^2 = 64 + 36 = 100$

$\boxed{r = 10}$



$\sin\theta = \frac{y}{r} = \frac{-6}{10} = -\frac{3}{5}$

$\cos\theta = \frac{8}{10} = \frac{4}{5}$

$\tan\theta = \frac{-6}{8} = \frac{-3}{4}$

$\cot\theta = \frac{-8}{6} = \frac{-4}{3}$

$\csc\theta = \frac{10}{-6} = -\frac{5}{3}$

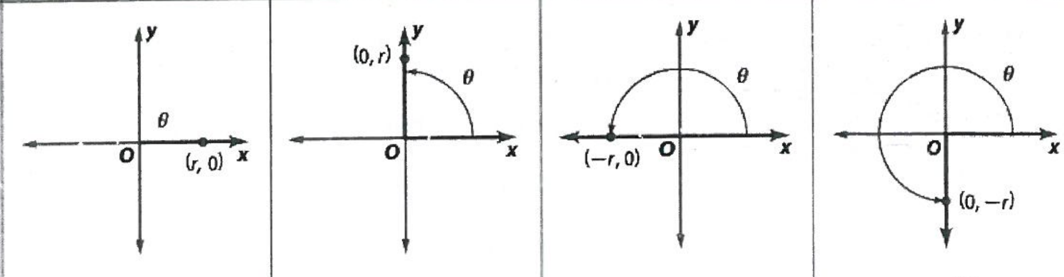
$\sec\theta = \frac{10}{8} = \frac{5}{4}$

Quadrantal Angles: When the terminal side of an angle θ that is in standard position lies on one of the coordinate axes, the angle is called a quadrantal angle. Now, we will find six trig functions when only θ is known.

StudyTip

Quadrantal Angles There are infinitely many quadrantal angles that are coterminal with the quadrantal angles listed at the right. The measure of a quadrantal angle is a multiple of 90° or $\frac{\pi}{2}$.

KeyConcept Common Quadrantal Angles



$\theta = 0°$ or 0 radians	$\theta = 90°$ or $\frac{\pi}{2}$ radians	$\theta = 180°$ or π radians	$\theta = 270°$ or $\frac{3\pi}{2}$ radians

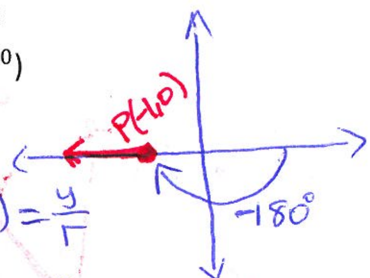
We can find the values of the trigonometric functions of quadrantal angles by chosing a point on the terminal side of the angle and evaluating the function at that point. We can pick any point, but we will choose r=1

Examples:

1.sin(-180⁰)

$y=0$ $r=1$

$x=-1$

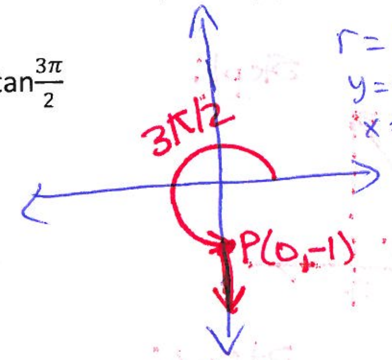


$\sin(-180°) = \frac{y}{r}$

$= \frac{0}{1}$

$= 0$

2. $\tan\frac{3\pi}{2}$



$r=1$

$y=-1$

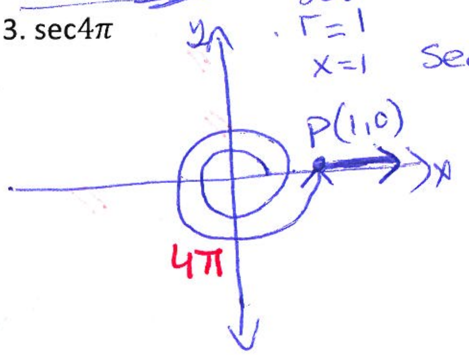
$x=0$

$\tan\frac{3\pi}{2} = \frac{y}{x}$

$= \frac{-1}{0}$

$=$ undefined

3. $\sec 4\pi$



$y=0$

$r=1$

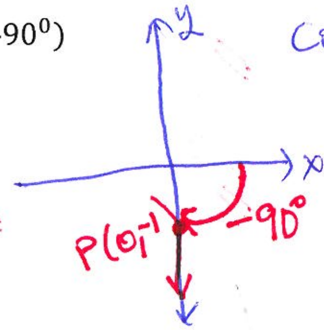
$x=1$

$\sec 4\pi = \frac{r}{x}$

$= \frac{1}{1}$

$= 1$

4. Cot(-90⁰)



$\cot(-90°) = \frac{x}{y}$

$= \frac{0}{-1}$

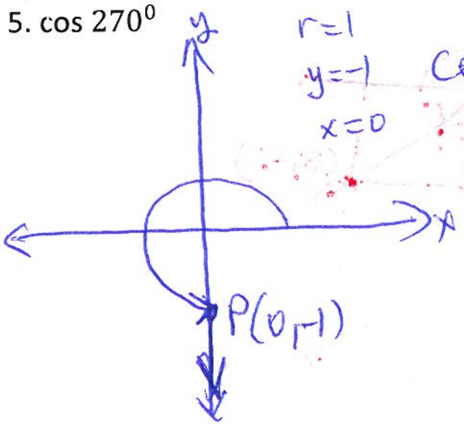
$= 0$

5. cos 270⁰

$r=1$

$y=-1$

$x=0$

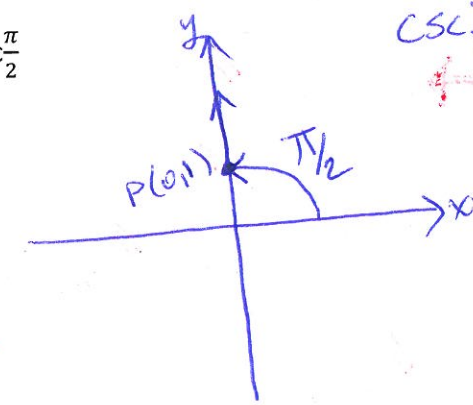


$\cos 270° = \frac{x}{r}$

$= \frac{0}{1}$

$= 0$

5. $\csc\frac{\pi}{2}$



$\csc\frac{\pi}{2} = \frac{r}{y}$

$= \frac{1}{1}$

$= 1$