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4-2 Study Guide and Intervention

Degrees and Radians

Angles and Their Measures One complete rotation can be represented by 360° or 2π radians. Thus, the following formulas can be used to relate **degree** and **radian** measures.

Degree/Radian Coversion Rules	
$1° = \frac{\pi}{180}$ radians	1 radian $= \left(\frac{180}{\pi}\right)°$

If two angles have the same initial and terminal sides, but different measures, they are called **coterminal angles**.

Example **Write each degree measure in radians as a multiple of π and each radian measure in degrees.**

a. 36°

$36° = 36°\left(\frac{\pi \text{ radians}}{180°}\right)$ Multiply by $\frac{\pi \text{ radians}}{180°}$.

$= \frac{\pi}{5}$ radians or $\frac{\pi}{5}$ Simplify.

b. $-\frac{17\pi}{3}$

$-\frac{17\pi}{3} = -\frac{17\pi}{3}$ radians Multiply by $\frac{180°}{\pi \text{ radians}}$.

$= -\frac{17\pi}{3}$ radians $\left(\frac{180°}{\pi \text{ radians}}\right) = -1020°$ Simplify.

Exercises

Write each degree measure in radians as a multiple of π and each radian measure in degrees.

1. $-250° = -250°\left(\frac{\pi}{180}\right)$ $= -\frac{25\pi}{18}$
2. $6° = 6°\left(\frac{\pi}{180°}\right) = \frac{\pi}{30}$
3. $-145° = -145°\left(\frac{\pi}{180°}\right)$ $= -29\pi/36$
4. $870° =$ $29\pi/6$
5. $18° = \pi/10$
6. $-820° = -\frac{41\pi}{9}$
7. 4π $= 720°$
8. $\frac{13\pi}{30} = 78°$
9. ~~9.~~ -1
10. $\frac{3\pi}{16} = 33.75°$
11. ~~11.~~ -2.56
12. $-\frac{7\pi}{9} = -140°$

Identify all angles that are coterminal with the given angle.

13. $-\frac{\pi}{2}$ $-\frac{\pi}{2} + 2n\pi$
14. 135° $135° + 360n$
15. $\frac{5\pi}{3}$ $\frac{5\pi}{3} + 2n\pi$

4-2 Study Guide and Intervention *(continued)*

Degrees and Radians

Applications with Angle Measure The rate at which an object moves along a circular path is called its **linear speed**. The rate at which the object rotates about a fixed point is called its **angular speed**.

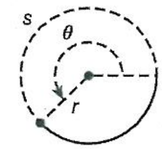
Suppose an object moves at a constant speed along a circular path of radius r.

If s is the arc length traveled by the object during time t, then the object's *linear speed* v is given by

$$v = \frac{s}{t}.$$

If θ is the angle of rotation (in radians) through which the object moves during time t, then the *angular speed* ω of the object is given by

$$\omega = \frac{\theta}{t}.$$



Example **Determine the angular speed and linear speed if 8.2 revolutions are completed in 3 seconds and the distance from the center of rotation is 7 centimeters. Round to the nearest tenth.**

The angle of rotation is $8.2 \times 2\pi$ or 16.4π radians.

$\omega = \frac{\theta}{t}$	Angular speed
$= \frac{16.4\pi}{3}$	$\theta = 16.4\pi$ radians and $t = 3$ seconds
≈ 17.17403984	Use a calculator.

Therefore, the angular speed is about 17.2 radians per second.

The linear speed is $\frac{r\theta}{t}$.

$v = \frac{s}{t}$	Linear speed
$= \frac{r\theta}{t}$	$s = r\theta$
$= \frac{7(16.4\pi)}{3}$	$r = 7$ centimeters, $\theta = 16.4\pi$ radians, and $t = 3$ seconds
$= 120.218278877$	Use a calculator.

Therefore, the linear speed is about 120.2 centimeters per second.

Exercises

Find the rotation in revolutions per minute given the angular speed and the radius given the linear speed and the rate of rotation.

rad $= \frac{1r}{2} \frac{v}{\pi}$

1. $\omega = 2.7$ rad/s   1rev = 2π rad

$= 2.7 \frac{rad}{sec}\left(\frac{1 sec}{1/60 min}\right) = \frac{2.7}{2\pi} 60 \frac{rev}{min}$

$=$ ≈ 25.8 rev/min

2. $\omega = \frac{4}{3}\pi$ rad/hr $= \frac{4}{3}\pi\left(\frac{1 rev}{2\pi}\right)\frac{1}{hr}\left(\frac{1 hr}{60 min}\right)$

$= 0.01$ rev/min

$v = r\frac{\theta}{t}$

3. $\omega = \frac{3}{2}\pi$ rad/min

$= \frac{3}{2}\pi \frac{1 rev}{2\pi min} = \frac{3}{4}$ rev/min

4. $v = 24.8$ m/s, 120 rev/min $v = r\frac{\theta}{t} = r\omega$

$24.8 \frac{m}{s} = r \cdot 120 \frac{rev}{min} \cdot \left(\frac{2\pi}{1 rev}\right)\left(\frac{1 min}{60 s}\right)$

2m = r

$v = r\omega$

5. $v = 118$ ft/min, 3.6 rev/s $= \omega$

$v = r\omega \Rightarrow 118 \frac{ft}{min}\left(\frac{1 min}{60 sec}\right) = r \cdot 3.6 \frac{rev}{sec}\left(\frac{2\pi}{1 rev}\right)$

$\frac{118 ft}{3.6 \times 2 \times 3.14} = \frac{r \times 3.6 \times 2\pi}{3.6 \times 2 \times 3.14}$

$0.087 = r$

r = 0.09 ft

6. $v = 256$ in./h, 0.5 rev/min

$v = r\omega$

$256 \frac{in}{hr}\left(\frac{1 hr}{60 min}\right) = r \cdot \left(0.5 \frac{rev}{min}\right)\left(\frac{2\pi}{1 rev}\right)$

$\frac{256}{3.14 \times 60 \times 0.5 \times 2} = r \Rightarrow \boxed{r = 1.4 \text{ in}}$

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4-3 Study Guide and Intervention

Trigonometric Functions on the Unit Circle

Trigonometric Functions of Any Angle

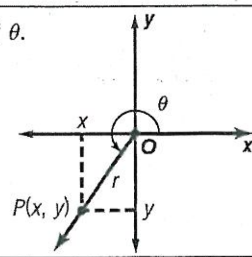
The definitions of the six trigonometric functions may be extended to include any angle as shown below.

Let θ be any angle in standard position and point $P(x, y)$ be a point on the terminal side of θ. Let r represent the nonzero distance from P to the origin. That is, let $r = \sqrt{x^2 + y^2} \neq 0$. Then the trigonometric functions of θ are as follows.

$\sin \theta = \frac{y}{r}$ $\qquad$ $\csc \theta = \frac{r}{y}, y \neq 0$

$\cos \theta = \frac{x}{r}$ $\qquad$ $\sec \theta = \frac{r}{x}, x \neq 0$

$\tan \theta = \frac{y}{x}, x \neq 0$ $\qquad$ $\cot \theta = \frac{x}{y}, y \neq 0$



Solve this page using reference angles

You can use the following steps to find the value of a trigonometric function of any angle θ.

1. Find the reference angle θ'.
2. Find the value of the trigonometric function for θ'.
3. Use the quadrant in which the terminal side of θ lies to determine the sign of the trigonometric function value of θ.

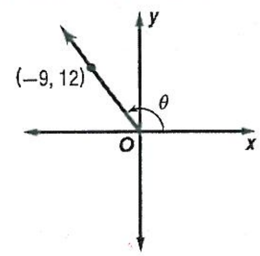
Example **Let (−9, 12) be a point on the terminal side of an angle θ in standard position. Find the exact values of the six trigonometric functions of θ.**

Use the values of x and y to find r.

$r = \sqrt{x^2 + y^2}$ $\qquad$ Pythagorean Theorem

$= \sqrt{(-9)^2 + 12^2}$ $\qquad$ $x = -9$ and $y = 12$

$= \sqrt{225}$ or 15 $\qquad$ Take the positive square root.



Use $x = -9$, $y = 12$, and $r = 15$ to write the six trigonometric ratios.

$\sin \theta = \frac{y}{r} = \frac{12}{15}$ or $\frac{4}{5}$ $\qquad$ $\cos \theta = \frac{x}{r} = \frac{-9}{15}$ or $-\frac{3}{5}$ $\qquad$ $\tan \theta = \frac{y}{x} = \frac{12}{-9}$ or $-\frac{4}{3}$

$\csc \theta = \frac{r}{y} = \frac{15}{12}$ or $\frac{5}{4}$ $\qquad$ $\sec \theta = \frac{r}{x} = \frac{15}{-9}$ or $-\frac{5}{3}$ $\qquad$ $\cot \theta = \frac{x}{y} = \frac{-9}{12}$ or $-\frac{3}{4}$

Exercises

The given point lies on the terminal side of an angle θ in standard position. Find the values of the six trigonometric functions of θ.

1. (2, −5)

$r = \sqrt{29}$

$\sin\theta = \frac{-5}{\sqrt{29}} = \frac{-5\sqrt{29}}{29}$

$\cos\theta = \frac{2\sqrt{29}}{29}$

$\tan\theta = -\frac{5}{2}$, $\cot\theta = \frac{-2}{5}$

$\sec\theta = \frac{\sqrt{29}}{2}$

$\csc\theta = -\frac{\sqrt{29}}{5}$

2. (12, 4)

$r = \sqrt{16 \times 10} = 4\sqrt{10}$

$\sin\theta = \frac{4}{4\sqrt{10}} = \frac{\sqrt{10}}{10}$, $\sec\theta = \frac{\sqrt{10}}{3}$

$\cos\theta = \frac{12}{4\sqrt{10}} = \frac{3\sqrt{10}}{10}$, $\csc\theta = \sqrt{10}$

$\tan\theta = \frac{4}{12} = \frac{1}{3}$

$\cot\theta = 3$

3. (−3, −8)

$r = \sqrt{73}$

$\sin\theta = \frac{-8\sqrt{73}}{73}$, $\cos\theta = \frac{-3\sqrt{73}}{73}$

$\tan\theta = \frac{8}{3}$, $\cot\theta = \frac{3}{8}$

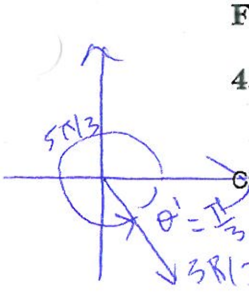
$\sec\theta = \frac{-\sqrt{73}}{3}$

$\csc\theta = \frac{-\sqrt{73}}{8}$

Find the exact value of each expression.

4. $\sin \frac{5\pi}{3} = -\frac{\sqrt{3}/2}{1} = -\frac{\sqrt{3}}{2}$

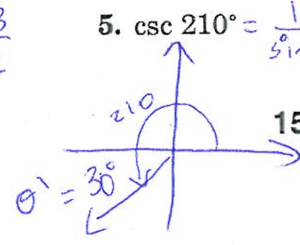
$\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$



5. $\csc 210° = \frac{1}{\sin 210°} = \frac{1}{-1/2} = -2$

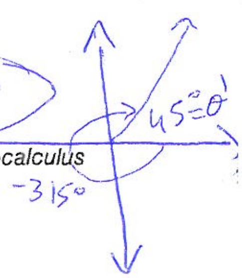
$\csc 210° = \csc 30°$

$= \frac{1}{\sin 30°} = \frac{1}{-1/2} = -2$



6. $\cot(-315°)$

$\cot 45° = 1$



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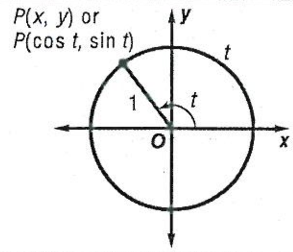
4-3 Study Guide and Intervention *(continued)*

Trigonometric Functions on the Unit Circle

Trigonometric Functions on the Unit Circle You can use the unit circle to find the values of the six trigonometric functions for θ. The relationships between θ and the point $P(x, y)$ on the unit circle are shown below.

Let t be any real number on a number line and let $P(x, y)$ be the point on t when the number line is wrapped onto the unit circle. Then the trigonometric functions of t are as follows.

$\sin t = y$	$\cos t = x$	$\tan t = \frac{y}{x}, x \neq 0$
$\csc t = \frac{1}{y}, y \neq 0$	$\sec t = \frac{1}{x}, x \neq 0$	$\cot t = \frac{x}{y}, y \neq 0$



Therefore, the coordinates of P corresponding to the angle t can be written as $P(\cos t, \sin t)$.

Example **Find the exact value of tan $\frac{5\pi}{3}$. If undefined, write *undefined*.**

$\frac{5\pi}{3}$ corresponds to the point $(x, y) = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ on the unit cirle.

$\tan t = \frac{y}{x}$ Definition of tan t

$\tan \frac{5\pi}{3} = \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}}$ $x = \frac{1}{2}$ and $y = -\frac{\sqrt{3}}{2}$ when $t = \frac{5\pi}{3}$

$\tan \frac{5\pi}{3} = -\sqrt{3}$ Simplify.

Exercises

Find the exact value of each expression. If undefined, write *undefined*.

1. $\tan \frac{\pi}{2} = \frac{y}{x} = \frac{1}{0} =$ undefined

2. $\sec -\frac{3\pi}{4} = \frac{1}{x} = \frac{1}{-\frac{\sqrt{2}}{2}} = \frac{-2}{\sqrt{2}} = \frac{-2\sqrt{2}}{2} = -\sqrt{2}$

3. $\cos \frac{7\pi}{6} = \frac{x}{r} = \frac{-\sqrt{3}}{2}$

4. $\sin \frac{5\pi}{4} = \frac{-\sqrt{2}}{2}$

5. $\cot \frac{4\pi}{3} = \frac{x}{y} = \frac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \frac{\sqrt{3}}{3}$

6. $\csc -\frac{5\pi}{3} = \frac{2\sqrt{3}}{3}$

7. $\tan -60° = -\sqrt{3}$

8. $\cot 270° = 0$