Name________________________________ Date_____________ Period________

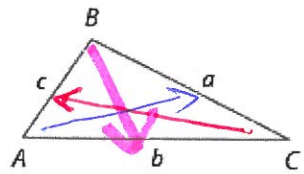
Objective: I can solve oblique triangles by using the Law of Sines and the Law of Cosines.

4.7 The Law of Sines and the Law of Cosines

The law of sines and the law of cosines are used to solve oblique triangles – triangles that are not right triangles.

KeyConcept Law of Sines

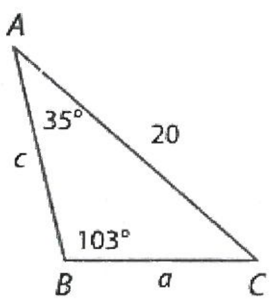
If $\triangle ABC$ has side lengths a, b, and c representing the lengths of the sides opposite the angles with measures A, B, and C, then $\boxed{\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}}$.



You can apply the **Law of Sines** to solve an oblique triangle if you know the measures of two angles and a nonincluded side (AAS), two angles and the included side (ASA), or two sides and a nonincluded angle (SSA).

Examples: Solve each triangle. Round side lengths to the nearest tenth, angle measures to the nearest degree.

1.



$\angle C = 180° - (35° + 103°)$

$\angle C = 42°$

$\dfrac{\sin 103°}{20} = \dfrac{\sin 35°}{a}$

$\dfrac{a(\sin 103°)}{\sin 103°} = \dfrac{20(\sin 35°)}{\sin 103°}$

$\boxed{a \approx 12}$

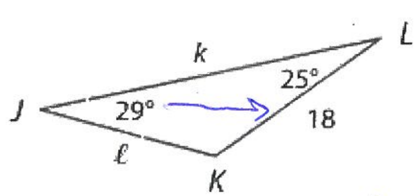
$\dfrac{\sin 103°}{20} = \dfrac{\sin 42°}{c}$

$c(\sin 103°) = 20(\sin 42°)$

$c = \dfrac{20(\sin 42°)}{\sin 103°}$

$c \approx 13.7 \approx 14$

2.



$\angle K = 180° - (29° + 25°)$

$\angle K = 126°$

$\dfrac{\sin 29°}{18} = \dfrac{\sin 25°}{\ell}$

$\ell(\sin 29°) = 18(\sin 25°)$

$\ell = \dfrac{18(\sin 25°)}{\sin 29°}$

$\ell = 15.7$

$\boxed{\ell \approx 16}$

$\dfrac{\sin 29°}{18} = \dfrac{\sin 126°}{k}$

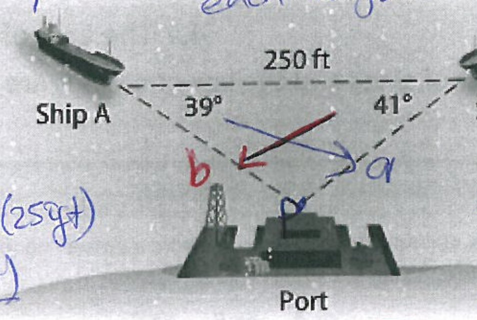
$k(\sin 29°) = 18(\sin 126°)$

$k = \dfrac{18(\sin 126°)}{\sin 29°}$

$\boxed{k = 30}$

3. Two ships are 250 ft apart and travelling to the same port as shown. Find the distance from the port to each ship.

First, identify opposite sides for each angle.



$\angle P = 180^\circ - (39^\circ + 41^\circ)$

$\boxed{\angle P = 100^\circ}$

$$* \quad \boxed{\frac{\sin 39^\circ}{a} = \frac{\sin 41^\circ}{b} = \frac{\sin 100^\circ}{250\text{ft}}}$$

$$\frac{\sin 39^\circ}{a} = \frac{\sin 100^\circ}{250\text{ft}}$$

$$a(\sin 100^\circ) = (\sin 39^\circ)(250\text{ft})$$

$$a = \frac{(\sin 39^\circ)(250\text{ft})}{\sin 100^\circ}$$

$$a \approx 159.8$$

$$\boxed{a \approx 160\text{ft}}$$

Then, using *,

$$\frac{\sin 41^\circ}{b} = \frac{\sin 100^\circ}{250\text{ft}}$$

$$b \sin 100^\circ = (\sin 41^\circ)(250\text{ft})$$

$$b = \frac{(\sin 41^\circ)(250\text{ft})}{\sin 100^\circ}$$

$$b = 166.\overline{6}$$

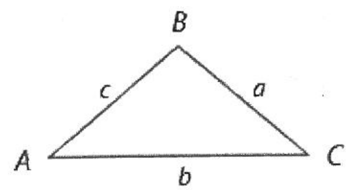
$$\boxed{b \approx 167\text{ft}}$$

You can use the **Law of Cosines** to solve an oblique triangle for the remaining two cases: when you are given the measures of three sides (SSS) or the measures of two sides and their included angle (SAS).

KeyConcept Law of Cosines

In $\triangle ABC$, if sides with lengths a, b, and c are opposite angles with measures A, B, and C, respectively, then the following are true.

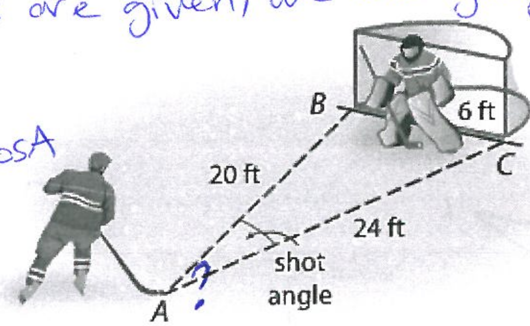
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cos C$$



Examples:

HOCKEY When a hockey player attempts a shot, he is 20 feet from the left post of the goal and 24 feet from the right post, as shown. If a regulation hockey goal is 6 feet wide, what is the player's shot angle to the nearest degree?

Since 3 side lengths are given, we are going to use Law of Cosines



$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$6^2 = 24^2 + 20^2 - 2(24)(20)\cos A$$

$$36 = 576 + 400 - 960\cos A$$

$$\frac{-940}{-960} = \frac{-960\cos A}{-960}$$

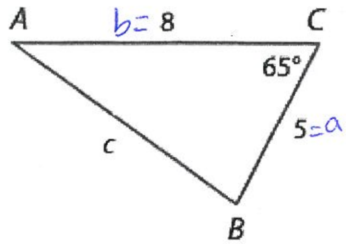
$$\frac{940}{960} = \cos A$$

$$\cos^{-1}\left(\frac{940}{960}\right) = A$$

$$\boxed{11.7^\circ = A}$$

So, the player's shoot angle is about $\underline{12^\circ}$

2. Find side length AB. Round side lengths to the nearest tenth and angle measures to nearest degree.



Using Law of Cosines

$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 5^2 + 8^2 - 2\cdot 5\cdot 8\cos 65°$

$c^2 = 25 + 64 - 80\cos 65°$

$c^2 = 89 - 80(0.423)$

$\sqrt{c^2} = \sqrt{89 - 33.8}$

$\boxed{c \cong 7.4}$

Using Law of Sines, you can find ∠A & ∠B

$\frac{\sin 65°}{7.4} = \frac{\sin A}{5}$

$\sin^{-1}\left(\frac{5\sin 65°}{7.4}\right) = A$

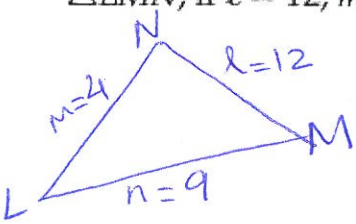
$\boxed{\angle A = 38°}$

$\angle B = 180° - (65° + 38°)$

$\boxed{\angle B = 77°}$

3. Find each angle measure for the following triangle.

$\triangle LMN$, if $\ell = 12$, $m = 4$, and $n = 9$



$l^2 = m^2 + n^2 - 2mn\cos L$

$12^2 = 4^2 + 9^2 - 2\times 4\times 9\cos L$

$144 = 16 + 81 - 72\cos L$

$-97 \quad -97$

$\frac{47}{-72} = \frac{-72\cos L}{-72}$

$\frac{-47}{72} = \cos L$

$\angle L = \cos^{-1}\left(\frac{-47}{72}\right)$

$\angle L = 130.75° \cong \boxed{131°}$

$m^2 = l^2 + n^2 - 2ln\cos M$

$4^2 = 12^2 + 9^2 - 2\times 12\times 9\cos M$

$16 = 144 + 81 - 216\cos M$

$-225 \quad -225$

$\frac{-209}{-216} = \frac{-216\cos M}{-216}$

$\left(\frac{209}{216}\right) = \cos M$

$\cos^{-1}\left(\frac{209}{216}\right) = \angle M$

$14.63° = \angle M$

$\boxed{\angle M \cong 15°}$

$\angle N = 180° - (131° + 15°)$

$\angle N = 180° - 146°$

$\boxed{\angle N = 34°}$

KeyConcept Area of a Triangle Given SAS

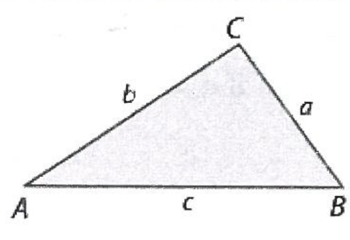
Words The area of a triangle is one half the product of the lengths of two sides and the sine of their included angle.

Symbols

Area $= \frac{1}{2}bc\sin A$

Area $= \frac{1}{2}ac\sin B$

Area $= \frac{1}{2}ab\sin C$



Examples:

1. $\triangle PQR$, if $P = 73°$, $q = 7$, and $r = 15$

$A = \frac{1}{2}qr\sin P$

$A = \frac{1}{2}(7)(15)\sin 73°$

$\boxed{A \cong 5,272 \text{ unit}^2}$

2. $\triangle ABC$, if $A = 42°$, $b = 12$, and $c = 19$

$A = \frac{1}{2}bc\sin A$

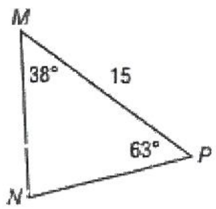
$A = \frac{1}{2}(12)(19)\sin 42°$

$\boxed{A \cong 76.3 \text{ unit}^2}$

Homework Due______________

Solve each triangle. Round to the nearest tenth if necessary.

1.



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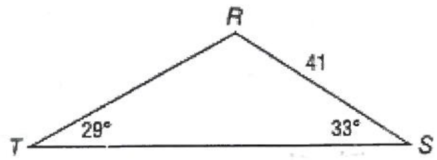

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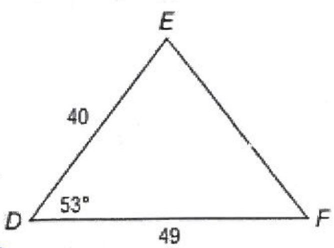

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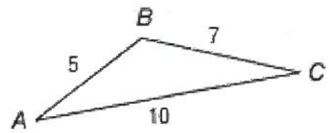
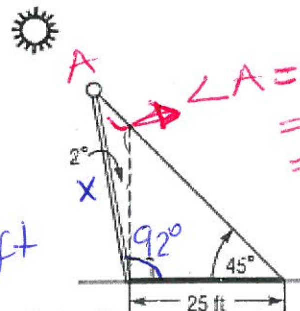


Warm Up

5. STREET LIGHTING A lamp post tilts toward the Sun at a 2° angle from the vertical and casts a 25-foot shadow. The angle from the tip of the shadow to the top of the lamp post is 45°. Find the length of the lamp post.

$$\frac{\sin 43^\circ}{25ft} = \frac{\sin 45^\circ}{x}$$

$$x = \frac{25(\sin 45^\circ)}{\sin 43^\circ} = \frac{17.677}{0.682} \approx 26ft$$



$\angle A = 180^\circ - (45^\circ + 92^\circ)$
$= 180^\circ - 137^\circ$
$= 43^\circ$

Use Heron's Formula to find the area of each triangle. Round to the nearest tenth.

6. $\triangle ABC$ if $a = 5$ ft, $b = 12$ ft, $c = 13$ ft

7. $\triangle FGH$ if $f = 11$ in., $g = 13$ in., $h = 16$ in.

8. $\triangle MNP$ if $m = 8$ yd, $n = 3.6$ yd, $p = 5.2$ yd

9. $\triangle XYZ$ if $x = 12$ cm, $y = 10$ cm, $z = 15.8$ cm

Find the area of each triangle to the nearest tenth.

10. $\triangle RST$ if $R = 115^\circ$, $s = 15$ yd, $t = 20$ yd

11. $\triangle MNP$ if $n = 4$ ft, $P = 69^\circ$, $N = 37^\circ$

12. $\triangle DEF$ if $d = 2$ ft, $E = 85^\circ$, $F = 19^\circ$

13. $\triangle JKL$ if $j = 68$ cm, $l = 110$ cm, $K = 42.5^\circ$

* Remember to use Law of Sines when you know AAS, ASA and SSA. And Use Law of Cosines when you were given SSS or SAS.