Name______________________________ Date____________ Period________

Identity Matrix: It is a matrix used in matrix multiplication such that when multiplied by another matrix, the resulting matrix is equal to the original matrix. You may call it as non-effective matrix.

$$I_1 = [1],\ I_2 = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\ I_3 = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix},\ \cdots,\ I_n = \begin{bmatrix}1 & 0 & 0 & \cdots & 0\\ 0 & 1 & 0 & \cdots & 0\\ 0 & 0 & 1 & \cdots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & 0 & \cdots & 1\end{bmatrix}$$

$$\text{if } A = \begin{bmatrix}a & b\\ c & d\end{bmatrix}, \text{ then } I = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \text{ such that } \begin{bmatrix}a & b\\ c & d\end{bmatrix}\cdot\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} = \begin{bmatrix}a & b\\ c & d\end{bmatrix}$$

***Therefore, **IA = A and AI = A**

Inverse Matrix: Two matrices are inverses of each other, if their product is the identity matrix.

$$\text{if } = \begin{bmatrix}a & b\\ c & d\end{bmatrix},\ \text{then } A^{-1} = \frac{1}{ad-bc}\begin{bmatrix}d & -b\\ -c & a\end{bmatrix} \text{ where } ad\text{-}bc \neq 0$$

***Therefore, $\boldsymbol{AA^{-1} = I \text{ or } A^{-1}A = I}$

Example: Find the inverse of matrix D

$$D = \begin{bmatrix}\overset{a}{2} & \overset{b}{-1}\\ \underset{c}{3} & \underset{d}{4}\end{bmatrix} \rightarrow D^{-1} = \frac{1}{ad-bc}\begin{bmatrix}d & -b\\ -c & a\end{bmatrix} = \frac{1}{2\cdot4-(-1)\cdot3}\begin{bmatrix}4 & 1\\ -3 & 2\end{bmatrix} = \frac{1}{11}\begin{bmatrix}4 & 1\\ -3 & 2\end{bmatrix}$$

Example: Determine whether matrices X and Y are inverses of each other.

If the product of X·Y is equal to identity matrix, then they are inverses

$$X = \begin{bmatrix}2 & 2\\ -1 & 4\end{bmatrix} \text{ and } Y = \begin{bmatrix}\frac{1}{2} & \frac{1}{2}\\ -1 & \frac{1}{4}\end{bmatrix}$$

$$X\cdot Y = \begin{bmatrix}2\cdot\frac{1}{2}+2(-1) & 2\cdot\frac{1}{2}+2\cdot\frac{1}{4}\\ (-1)\frac{1}{2}+4(-1) & (-1)\frac{1}{2}+4\cdot\frac{1}{4}\end{bmatrix} = \begin{bmatrix}-1 & 3/2\\ -9/2 & 1/2\end{bmatrix}$$

***The product of X and Y is not equal to identity matrix, so, they are not inverses.

Example: Determine whether matrices P and Q are inverses of each other.

$$P = \begin{bmatrix}3 & 4\\ 1 & 2\end{bmatrix} \text{ and } Q = \begin{bmatrix}1 & -2\\ -\frac{1}{2} & \frac{3}{2}\end{bmatrix}$$

$$P.Q = \begin{bmatrix}3\cdot1+4(-\frac{1}{2}) & 3(-2)+4\cdot\frac{3}{2}\\ 1.1+2.(-\frac{1}{2}) & 1(-2)+2.3/2\end{bmatrix} = \begin{bmatrix}3-2 & -6+6\\ 1-1 & -2+3\end{bmatrix} = \begin{bmatrix}1 & \\ 0 & \end{bmatrix}$$

***The product of P and Q is equal to identity matrix, therefore, P & Q are inverses of each other

*If the value of the determinant is zero, the matrix cannot have an inverse

Ex: $R = \begin{bmatrix}-4 & -3\\ 8 & 6\end{bmatrix}$ $R^{-1} = \frac{1}{-4\cdot6-(-3)8}\begin{bmatrix} & \\ & \end{bmatrix}$

Determinant = 0

R^{-1} cannot exist

Ex: $P = \begin{bmatrix}3 & 4\\ 5 & 2\end{bmatrix}$ $P^{-1} = \frac{1}{3.2-4.5}\begin{bmatrix}2 & -4\\ -5 & 3\end{bmatrix}$

$P^{-1} = \frac{-1}{+14}\begin{bmatrix}2 & -4\\ -5 & 3\end{bmatrix}$

Determinant is not equal to zero

Solving System of Linear Equations Using Inverses:

Invertible Square Linear Systems: If AX=B and A is invertible, then $X = A^{-1}B$

Examples: *Use an inverse matrix to solve the system of equations, if possible.*

1. $\begin{aligned} 2x - 3y &= -1 \\ -3x + 5y &= 3 \end{aligned}$

$\underbrace{\begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix}}_{\text{Inverse}} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$

$\frac{1}{5 \cdot 2-(-3)(-3)} \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{1} \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 3 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$ $\quad x=4$, $y=3$ $\quad (4,3)$

2. $\begin{aligned} 6x + y &= -8 \\ -4x - 5y &= -12 \end{aligned}$

$\underbrace{\begin{bmatrix} 6 & 1 \\ -4 & -5 \end{bmatrix}}_{\text{inverse}} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -8 \\ -12 \end{bmatrix}$

$\frac{1}{-30-(-4) \cdot 1} \begin{bmatrix} -5 & -1 \\ 4 & 6 \end{bmatrix} = \begin{bmatrix} 5/26 & 1/26 \\ -2/13 & -3/13 \end{bmatrix}$

$\begin{bmatrix} 5/26 & 1/26 \\ -2/13 & -3/13 \end{bmatrix} \cdot \begin{bmatrix} 6 & 1 \\ -4 & -5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5/26 & 1/26 \\ -2/13 & -3/13 \end{bmatrix} \cdot \begin{bmatrix} -8 \\ -12 \end{bmatrix} \rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ -4 \end{bmatrix}$

$x=-2$, $y=-4$ $\quad (-2,-4)$

3. $\begin{aligned} -4x - 2y &= -14 \\ -8x - 4y &= 4 \end{aligned}$

$\underbrace{\begin{bmatrix} -4 & -2 \\ -8 & -4 \end{bmatrix}}_{\text{Inverse}} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -14 \\ 4 \end{bmatrix}$

$\underbrace{\frac{1}{(-4)(-4)-(-2)(-8)}}_{\text{Determinant}=0} \begin{bmatrix} \quad \end{bmatrix}$

* So, the inverse doesn't exist. We cannot solve using inverse matrix.