

Determinants

Determinant is a square array of numbers or variables enclosed between two parallel lines. Every square matrix has a number associated with it called its determinant.

Second Order Determinant (for a 2X2 matrix): The difference of the products of the two diagonals

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc \quad *$$

Example 1:

$$\begin{vmatrix} -2 & 5 \\ 6 & 8 \end{vmatrix} = (-2)(8) - (5)(6) = -46$$

Example 2:

$$b. \begin{vmatrix} 7 & 4 \\ -3 & 2 \end{vmatrix} = (7)(2) - (4)(-3) = 26$$

Third-Order Determinant (for a 3X3 matrix): Expansion of minors can be used to evaluate this determinant.

Minor of an element is the determinant formed by deleting the row and column of containing element.

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix} \quad *$$

$$\text{Example: } \begin{vmatrix} 2 & 7 & -3 \\ -1 & 5 & -4 \\ 6 & 9 & 0 \end{vmatrix} = 2 \begin{vmatrix} 5 & -4 \\ 9 & 0 \end{vmatrix} - 7 \begin{vmatrix} -1 & -4 \\ 6 & 0 \end{vmatrix} + (-3) \begin{vmatrix} -1 & 5 \\ 6 & 9 \end{vmatrix}$$

$$= 2(0 - (-36)) - 7(0 - (-24)) - 3(-9 - 30) = 72 - 168 + 117 = 21$$

Examples: Find the value of each determinant given below.

$$1. \begin{vmatrix} 7 & 8 \\ 3 & -2 \end{vmatrix} = ad - bc = 7 \cdot (-2) - 3 \cdot 8$$

$$= -14 - 24$$

$$= -38$$

$$2. \begin{vmatrix} -3 & -6 \\ 4 & 8 \end{vmatrix} = (-3) \cdot 8 - 4(-6)$$

$$= -24 + 24$$

$$= 0$$

$$3. \begin{vmatrix} 1 & 6 & 4 \\ -2 & 3 & 1 \\ 1 & 6 & 4 \end{vmatrix} = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$$= 1 \begin{vmatrix} 3 & 1 \\ 6 & 4 \end{vmatrix} - 6 \begin{vmatrix} -2 & 1 \\ 1 & 4 \end{vmatrix} + 4 \begin{vmatrix} -2 & 3 \\ 1 & 6 \end{vmatrix}$$

$$= 1 \cdot (3 \cdot 4 - 6 \cdot 1) - 6 \cdot ((-2) \cdot 4 - 1 \cdot 1) + 4 \cdot ((-2) \cdot 6 - 3 \cdot 1)$$

$$= 6 + 54 - 60$$

$$= 0$$

$$4. \begin{vmatrix} -1 & 4 & 0 \\ 3 & -2 & -5 \\ -3 & -1 & 2 \end{vmatrix} = -1 \begin{vmatrix} -2 & -5 \\ -3 & 2 \end{vmatrix} - 4 \begin{vmatrix} 3 & -5 \\ -3 & 2 \end{vmatrix} + 0 \begin{vmatrix} 3 & -2 \\ -3 & -1 \end{vmatrix}$$

$$= -1 \cdot (-2 \cdot 2 - (-5) \cdot (-1)) - 4 \cdot (3 \cdot 2 - (-5) \cdot (-3)) + 0 \cdot (3 \cdot (-1) - (-2) \cdot (-3))$$

$$= -1 \cdot (-4 - 5) - 4 \cdot (6 - 15) + 0$$

$$= 9 + 36$$

$$= 45$$

Homework: Find the determinant of the following matrices.

Due: 4/23/15

$$A = \begin{bmatrix} -4 & 3 & 2 \\ 0 & -1 & 1 \\ -2 & 1 & 0 \end{bmatrix}$$

$$\begin{aligned} \det A &= -4 \begin{vmatrix} -1 & 1 \\ 1 & 0 \end{vmatrix} - 3 \begin{vmatrix} 0 & 1 \\ -2 & 1 \end{vmatrix} + 2 \begin{vmatrix} 0 & -1 \\ -2 & 1 \end{vmatrix} \\ &= -4(-1 \cdot 0 - 1 \cdot 1) - 3(0 \cdot 0 - (-2) \cdot 1) + 2(0 \cdot 1 - (-1) \cdot (-2)) \\ &= 4 - 6 - 4 = -6 \end{aligned}$$

$$B = \begin{bmatrix} 7 & -1 \\ 4 & 4 \end{bmatrix}$$

$$\begin{aligned} \det B &= 7 \cdot 4 - (-1) \cdot 4 \\ &= 28 - (-4) \\ &= 28 + 4 = 32 \end{aligned}$$

$$C = \begin{bmatrix} 4 & -3 \\ 5 & 2 \end{bmatrix}$$

$$\begin{aligned} \det C &= 4 \cdot 2 - 5 \cdot (-3) \\ &= 8 + 15 \\ &= 23 \end{aligned}$$

$$D = \begin{bmatrix} 2 & 4 \\ -3 & -6 \end{bmatrix}$$

$$\begin{aligned} \det D &= 2 \cdot (-6) - 4 \cdot (-3) \\ &= -12 + 12 = 0 \end{aligned}$$

$$\begin{aligned} \begin{bmatrix} -1 & 1 & 0 \\ 1 & -1 & 1 \\ 1 & -1 & 1 \end{bmatrix} &\rightarrow \det = -1 \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} - 1 \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} + 0 \begin{vmatrix} 1 & -1 \\ 1 & -1 \end{vmatrix} \\ &= -1(-1 \cdot 1 - (1) \cdot 1) - 1(1 \cdot 1 - 1 \cdot 1) + 0 \\ &= -1(-1 + 1) - (1 - 1) + 0 \\ &= 0 \end{aligned}$$