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Chapter 8 Study Guide

Distance formula $|\mathbf{v}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Magnitude $|\mathbf{v}| = \sqrt{a^2 + b^2}$.

Unit vector $\mathbf{u} = \dfrac{\mathbf{v}}{|\mathbf{v}|}$

Direction Angle $\mathbf{v} = \langle a, b \rangle = \langle |\mathbf{v}| \cos\theta, |\mathbf{v}| \sin\theta \rangle = |\mathbf{v}|(\cos\theta)\mathbf{i} + |\mathbf{v}|(\sin\theta)\mathbf{j}$

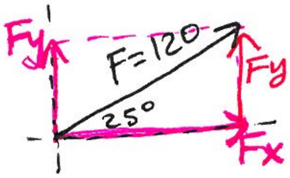
Dot Product $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2$.

Angle Between Two Vectors $\cos\theta = \dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|\,|\mathbf{b}|}$

Projection of u onto v $\text{proj}_{\mathbf{v}}\mathbf{u} = \left(\dfrac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{v}|^2}\right)\mathbf{v}$.

1. SLEDDING Jordyn pulls a sled with a force of 120 newtons at an angle of 25° with the horizontal. Find the magnitude of the horizontal component of the force.

A 50.7 N **B** 56.0 N **C** 88.3 N **D** 108.8 N



$\cos 25° = \dfrac{F_x}{120}$

$F_x = 120 \cdot \cos 25° = 108.7 \sim 109\text{N}$

2. Let $\overrightarrow{AB}$ be a vector with the given initial point $A(8, -4)$ and terminal point $B(-2, -3)$. Write $\overrightarrow{AB}$ as a linear combination of the vectors **i** and **j**.

F $10\mathbf{i} - \mathbf{j}$ **G** $6\mathbf{i} - 7\mathbf{j}$ **H** $-10\mathbf{i} + \mathbf{j}$ **J** $-6\mathbf{i} + 7\mathbf{j}$

$\overrightarrow{AB} = (-2-8, -3-(-4))$

$\overrightarrow{AB} = (-10, 1)$ $\boxed{\overrightarrow{AB} = -10i + j}$

3. Find the component form and magnitude of the vector with initial point $A(-6, 4)$ and terminal point $B(-2, -1)$.

A $\langle 4, -5 \rangle; \sqrt{41}$ **B** $\langle 4, -5 \rangle; 9$ **C** $\langle -4, 5 \rangle; \sqrt{41}$ **D** $\langle -4, 5 \rangle; 9$

$\overrightarrow{AB} = \langle -2-(-6), -1-4 \rangle$

$\boxed{\overrightarrow{AB} = \langle 4, -5 \rangle}$

$|AB| = \sqrt{4^2 + (-5)^2}$

$|AB| = \sqrt{16+25} = \boxed{\sqrt{41}}$

4. Find the dot product of $\mathbf{u} = \langle 8, 7 \rangle$ and $\mathbf{v} = \langle -3, -2 \rangle$. Then determine if **u** and **v** are orthogonal.

F −9, orthogonal
G −9, not orthogonal
H −38, not orthogonal
J −38, orthogonal

$u \cdot v = \langle 8, 7 \rangle \cdot \langle -3, -2 \rangle$

$= 8 \cdot (-3) + 7 \cdot (-2)$

$= -24 - 14 = \boxed{-38}$

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5. If **u** = ⟨–8, 7⟩ and **v** = ⟨4, –6⟩, find 2**u** – **v**.

A ⟨–20, 20⟩ **B** ⟨20, –20⟩ **C** ⟨–12, 8⟩ **D** ⟨12, –8⟩

$2u - v = 2\langle -8, 7\rangle - \langle 4, -6\rangle$

$= \langle -16, 14\rangle + \langle -4, 6\rangle$

$= \boxed{\langle -20, 20\rangle}$

6. Find the projection of **u** = ⟨7, –3⟩ onto **v** = ⟨4, 3⟩. Then write **u** as the sum of two orthogonal vectors, one of which is the projection of **u** onto **v**.

See the attached sheet for the solution

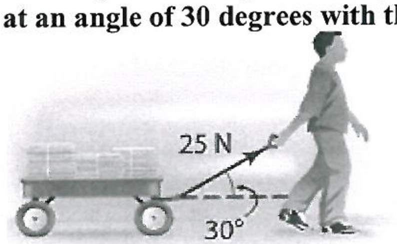
7. Find the measure of the angle θ between **u** = ⟨9, 9⟩ and **v** = ⟨–7, 8⟩ to the nearest tenth of a degree.

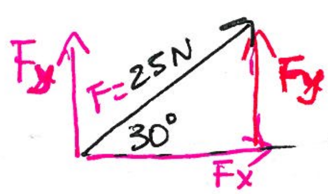
$\cos\theta = \dfrac{a \cdot b}{|a||b|} = \dfrac{\langle 9, 9\rangle \cdot \langle -7, 8\rangle}{(\sqrt{9^2+9^2})(\sqrt{(-7)^2+8^2})}$

$\cos\theta = \dfrac{9\cdot(-7) + 9\cdot 8}{(\sqrt{162})(\sqrt{130})}$

$\cos\theta = \dfrac{9}{145} \rightarrow \theta = \cos^{-1}\left(\dfrac{9}{145}\right) = 86.44^\circ \sim \boxed{86^\circ}$

8. Henry uses a wagon to carry newspapers for his paper route. He is pulling the wagon with a force of 25 Newtons at an angle of 30 degrees with the horizontal.





$\sin 30^\circ = \dfrac{Fy}{25}$

$\cos 30^\circ = \dfrac{Fx}{25}$

a) What is magnitude of the horizontal component of the force?
b) What is the magnitude of the vertical component of the force?

a) $\cos 30^\circ = \dfrac{Fx}{25}$

$Fx = 25 \cdot \cos 30^\circ$

$Fx = 21.65 \sim \boxed{22N}$

b) $\sin 30^\circ = \dfrac{Fy}{25}$

$Fy = 25 \sin 30^\circ$

$Fy = 12.5 \sim \boxed{13N}$

6) $u = \langle 7, -3 \rangle \quad v = \langle 4, 3 \rangle$

$$w_1 = \text{proj}_v u = \frac{u \cdot v}{|v|^2} v$$

$$w_1 = \frac{7 \cdot 4 + (-3) \cdot 3}{\left(\sqrt{4^2 + 3^2}\right)^2} \langle 4, 3 \rangle$$

$$w_1 = \frac{28 - 9}{\left(\sqrt{25}\right)^2} \langle 4, 3 \rangle$$

$$w_1 = \frac{19}{25} \langle 4, 3 \rangle$$

$$w_1 = \left\langle \frac{76}{25}, \frac{57}{25} \right\rangle$$

$$u = w_1 + w_2 \Rightarrow w_2 = u - w_1$$

$$w_2 = \langle 7, -3 \rangle - \left\langle \frac{76}{25}, \frac{57}{25} \right\rangle$$

$$w_2 = \left\langle \underset{(25)}{7} - \frac{76}{25}, \underset{(25)}{-3} - \frac{57}{25} \right\rangle$$

$$w_2 = \left\langle \frac{175}{25} - \frac{76}{25}, -\frac{75}{25} - \frac{57}{25} \right\rangle$$

$$w_2 = \left\langle \frac{99}{25}, \frac{-132}{25} \right\rangle$$

$$\boxed{u = w_1 + w_2 = \left\langle \frac{76}{25}, \frac{57}{25} \right\rangle + \left\langle \frac{99}{25}, \frac{-132}{25} \right\rangle}$$